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**GENERAL CERTIFICATE OF SECONDARY EDUCATION  
MATHEMATICS SYLLABUS A**

**J512/04**

Paper 4  
(Higher Tier)

Solutions

Candidates answer on the question paper

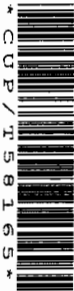
**OCR Supplied Materials:**  
None

**Other Materials Required:**

- Electronic calculator
- Geometrical instruments
- Tracing paper (optional)

**Wednesday 14 January 2009  
Afternoon**

**Duration: 2 hours**



<b>Candidate Forename</b>		<b>Candidate Surname</b>	
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<b>Centre Number</b>										<b>Candidate Number</b>					
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**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.

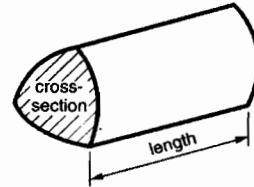
**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are expected to use an electronic calculator for this paper.
- Use the  $\pi$  button on your calculator or take  $\pi$  to be 3.142 unless the question says otherwise.
- The total number of marks for this paper is **100**.
- This document consists of **16** pages. Any blank pages are indicated.

<b>FOR EXAMINER'S USE</b>

## Formulae Sheet: Higher Tier

**Volume of prism** = (area of cross-section)  $\times$  length

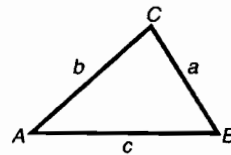


**In any triangle ABC**

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

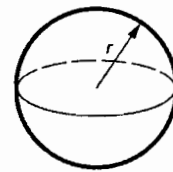
**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2} ab \sin C$



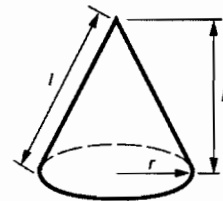
**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4\pi r^2$



**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



**The Quadratic Equation**

The solutions of  $ax^2 + bx + c = 0$ ,  
where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 Calculate.

(a)  $\frac{16.5}{8.25+5.15}$

Give your answer correct to 1 decimal place.

$$= \frac{16.5}{(8.25+5.15)}$$

$$= 1.2313 = 1.2 \text{ to 1 d.p.}$$

(a) 1.2 [2]

(b)  $\frac{45}{(0.3)^2}$

$$45 \div 0.3^2 = 500$$

(b) 500 [2]

2 Josh painted his bedroom.

Complete his paint bill by working out the three missing values.

$$3 \times £17.99 = £53.97$$

$$£77.95 - £53.97 = £23.98$$

$$£23.98 \div £11.99 = 2$$

Paint Bill		
3 tins silk emulsion	@ £17.99 per tin	£ <u>53.97</u>
<u>2</u> tins gloss	@ £11.99 per tin	£ <u>23.98</u>
<b>Total cost</b>		<b>£ 77.95</b>

[4]

3 (a) Factorise.

(i)  $6x + 16$

(a)(i)  $2(3x + 8)$  [1]

(ii)  $x^2 + 6x$

(ii)  $x(x + 6)$  [1]

(b) Solve.

(i)  $\frac{x}{12} = 6$

$x = 6 \times 12$

(b)(i)  $x = 72$  [1]

(ii)  $6x + 1 = 11 + 4x$

$6x - 4x = 11 - 1$

$2x = 10$

$x = \frac{10}{2}$

(ii)  $x = 5$  [3]

(iii)  $\frac{x}{6} + 2 = 9$

$x + 12 = 54$  (multiplying equation by 6)

$x = 54 - 12$

$x = 42$

(iii)  $x = 42$  [2]

(c) Rearrange the following to make  $x$  the subject.

$y = 6x - 7$

$y + 7 = 6x$

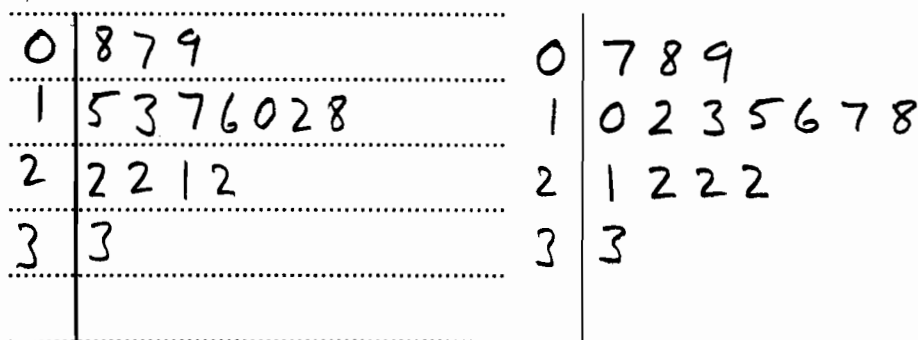
$\frac{y+7}{6} = x$

(c)  $x = \frac{y+7}{6}$  [2]

- 4 15 women each changed a car wheel.  
These are the times taken, in minutes.

22	15	13	17	22
8	16	21	7	10
12	33	9	18	22

- (a) Draw an ordered stem and leaf diagram to show these times.



Key:  $1|6 = 16$  minutes [3]

- (b) Work out the median and range of these times.

Median is 8<sup>th</sup> out of 15 which is 16

Range is  $33 - 7 = 26$

(b) Median = 16 minutes  
Range = 26 minutes [2]

15 men each changed a car wheel.  
The median time taken by these men was 16 minutes.  
The range of their times was 33 minutes.

- (c) Write down one comparison between the times taken by these men and women.

The men and the women had the same median time OR [1]

The women had a smaller range than the men

5 (a) The  $n$ th term of a sequence is  $n^2 + 2$ .

Write down the first three terms of this sequence.

$$1^2 + 2 = 1 + 2 = 3$$

$$2^2 + 2 = 4 + 2 = 6$$

$$3^2 + 2 = 9 + 2 = 11$$

(a) 3, 6, 11 [2]

(b) Another sequence begins

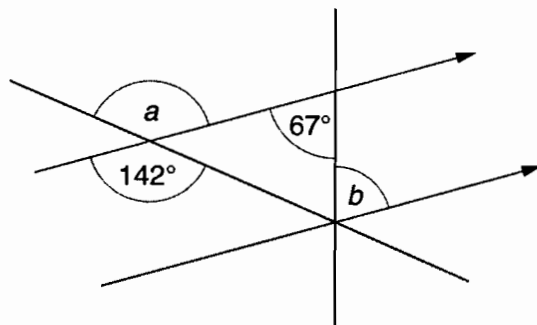
4	8	12	16	
7,	11,	15,	19,	.....

Write down the  $n$ th term of this sequence.

Adding 4 so  $4n$  involved in  $n^{\text{th}}$  term  
Adjustment of  $+3$  required

(b)  $n^{\text{th}}$  term =  $4n + 3$  [2]

6 (a) Find the sizes of angle  $a$  and angle  $b$ . (NOTE: DO NOT SAY  $n = 4n + 3$ )  
 Write down a reason for each answer.



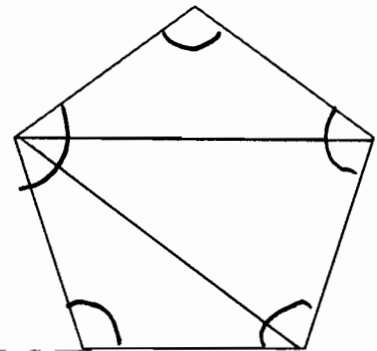
NOT TO SCALE

$a = 142$  ° Reason Vertically opposite angles are equal  
 $b = 67$  ° Reason alternate angles are equal [4]

(b) (i) The sum of the interior angles of a regular pentagon is  $540^\circ$ .

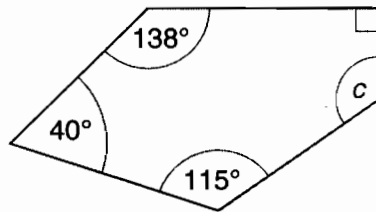
Without measuring any angles, explain why this is true.

Angles add up to  
same as 3  $\Delta$ s shown  
 $3 \times 180^\circ = 540^\circ$



[3]

- (ii) This is an irregular pentagon.



NOT TO SCALE

Still add up to  $540^\circ$ 

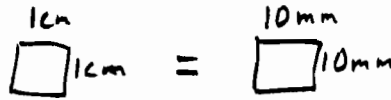
Work out angle c.

$$\begin{array}{r} 138 \\ 115 \\ \hline 253 \\ 90 \\ \hline 343 \\ 40 \\ \hline 383 \end{array}$$

$$540 - 383 = 157^\circ$$

(b)(ii) 157  $^\circ$  [2]

- (iii) The area of another pentagon is
- $4.5 \text{ cm}^2$
- .

Change  $4.5 \text{ cm}^2$  into  $\text{mm}^2$ .

$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$4.5 \text{ cm}^2 = 450 \text{ mm}^2$$

(iii) 450  $\text{mm}^2$  [2]

- (c) In the following expressions, the letters
- $f$
- ,
- $g$
- , and
- $h$
- represent lengths.

$fgh$

$f^2(g+h)$

$2f(g+h)$

Which one of these expressions could represent an area?

$fgh$

(c)  $2f(g+h)$  [1]

$L \times L \times L = \text{Vol}$

$f^2(g+h)$

$L^2(L+L) = L^2(L) = L^3 = \text{Vol}$

$2f(g+h) = L(L+L)$

$= L \times L = \text{Area} \checkmark$

7 (a)  $5x^3 = 40$ .

Work out the value of  $x$ .

$$x^3 = \frac{40}{5}$$

$$x^3 = 8$$

$$x = \sqrt[3]{8}$$

(a)  $x = 2$  [2]

(b) Write 52 as a product of prime factors.

$$\begin{array}{r} 2 \overline{)52} \\ 2 \overline{)26} \\ 13 \overline{)13} \\ 1 \end{array}$$

$$52 = 2 \times 2 \times 13 \quad \text{or} \quad 2^2 \times 13$$

(b)  $2 \times 2 \times 13$  [2]

(c) What is the LCM (least common multiple) of 27 and 33?

$$33 = 3 \times 11$$

$$\text{LCM} = 3 \times 3 \times 3 \times 11$$

$$27 = 3 \times 3 \times 3$$

$$= 297$$

(c)  $297$  [2]

(d) What is the HCF (highest common factor) of 96 and 144?

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \overline{)3} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \overline{)144} \\ 2 \overline{)72} \\ 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \overline{)3} \\ 1 \end{array}$$

(d)  $48$  [2]

$$96 = 2^5 \times 3$$

$$144 = 2^4 \times 3^2$$

$$\text{HCF} = 2^4 \times 3 = 48$$



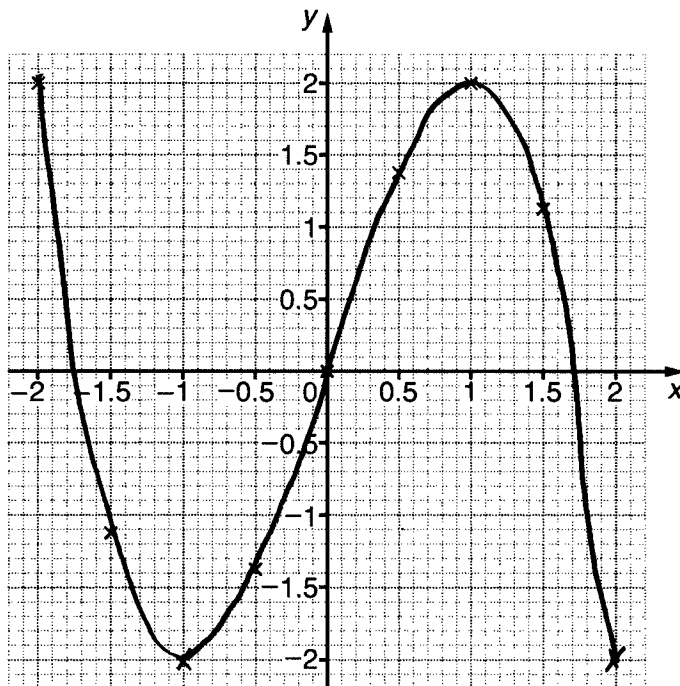
8 (a) Complete this table for  $y = 3x - x^3$ .

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	2	-1.125	-2	-1.375	0	1.375	2	1.125	-2

$$3(-1) - (-1)^3 = -3 + 1 = -2$$

(b) Complete the graph of  $y = 3x - x^3$  for  $-2 \leq x \leq 2$ .

$$3(2) - 2^3 = 6 - 8 = -2$$



(c) Use your graph to estimate the values of  $x$  when  $y = 1$ .

(c) -1.9, 0.3, 1.6 [2]

(d) Sam wants to use this graph to solve  $2x - x^3 = 0$ .

Find the equation of the line she should draw on the graph.

$$2x - x^3 = 0$$


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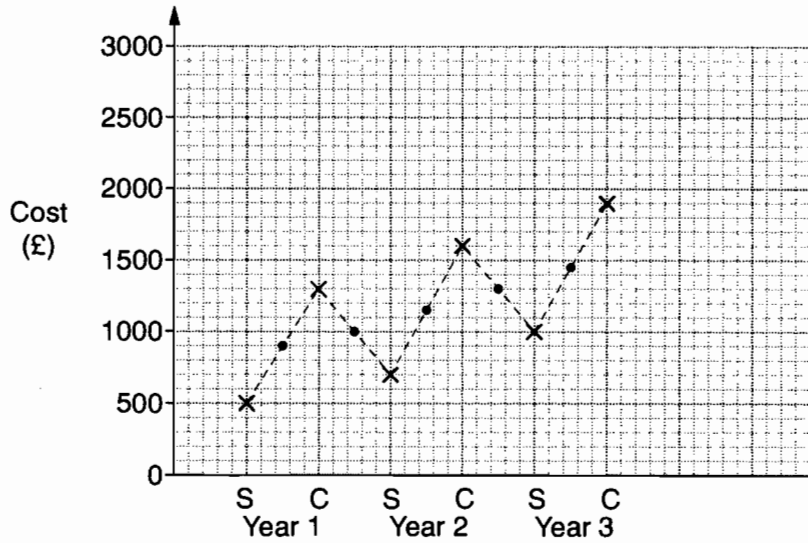

$$+ x \quad \quad \quad + x$$


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$$+ 3x - x^3 = x \quad \quad \quad \text{(d) } \underline{y = x} \quad \quad \quad [2]$$

Draw  $y = x$   
to go with  $y = 3x - x^3$

- 9 Barney kept a record of the cost, in £, of his office Summer parties (S) and Christmas parties (C). The graph shows these costs (x) and the 2-point moving averages (•) for three years.



- (a) Explain why Barney used 2-point moving averages.

2 parties per year [1]

- (b) Show how the first moving average has been calculated.

$$\frac{500 + 1300}{2} = \frac{1800}{2} = 900$$

- (c) The moving average calculated from the Year 3 Christmas and Year 4 Summer parties is £1420.

Calculate the cost of the Year 4 Summer party. *Let summer party be x*

$$\frac{1400 + x}{2} = 1420$$

$$1400 + x = 2840$$

$$x = 2840 - 1400$$

$$x = 1440$$

(c) £ 1440 [2]

- (d) 89 women and 31 men work for Barney.  
He wishes to take a representative sample, stratified by gender, of his staff.  
He decides on a sample of size 20.

How many women should Barney include in the sample?  $\text{Sampling fraction} = \frac{20}{120} = \frac{1}{6}$   
 $89 + 31 = 120 \text{ people}$

Women  $\frac{89}{6} = 14\frac{5}{6}$  Round to 15

(d) 15 [2]

- 10 Evan invested £50 in a savings account for 4 years at 6% compound interest per year.

He wants to use this formula to work out the amount, in £, in the savings account at the end of the 4 years.

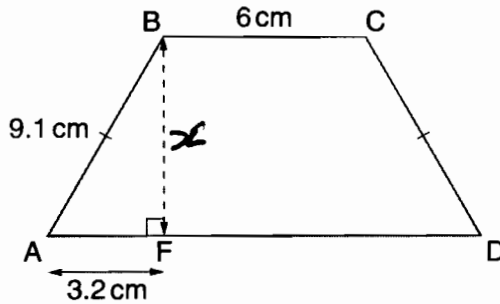
$$\text{Amount} = 50 \times c^d$$

$$50 \times 1.06^4$$

What values should he use for  $c$  and  $d$ ?

$c = \underline{1.06}$   $d = \underline{4}$  [3]

- 11 (a) ABCD is an isosceles trapezium.  
BF is perpendicular to AD.



NOT TO SCALE

- (i) Calculate BF. Let  $BF = x$

$$\begin{aligned} \text{Pythagoras } 3.2^2 + x^2 &= 9.1^2 \\ x^2 &= 9.1^2 - 3.2^2 = 72.57 \\ x &= \sqrt{72.57} = 8.5 \text{ cm to 1 d.p.} \end{aligned}$$

(a)(i) 8.5 cm [3]

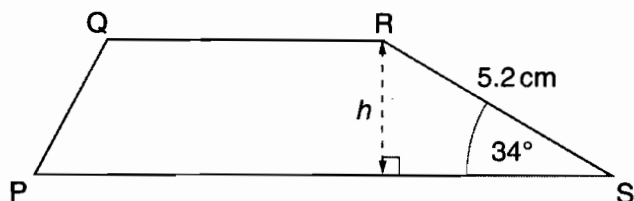
- (ii) Calculate the area of ABCD.

$$\begin{aligned} \text{Area} &= \frac{1}{2}(BC + AD) \times BF \\ \text{Isosceles trapezium so } AD &= 3.2 + 6 + 3.2 = 12.4 \end{aligned}$$

(ii) 78.2 cm<sup>2</sup> [3]

$$\text{Area} = \frac{1}{2}(6 + 12.4) \times 8.5 = 78.2 \text{ cm}^2$$

(b) PQRS is a trapezium.



NOT TO SCALE

Calculate  $h$ .

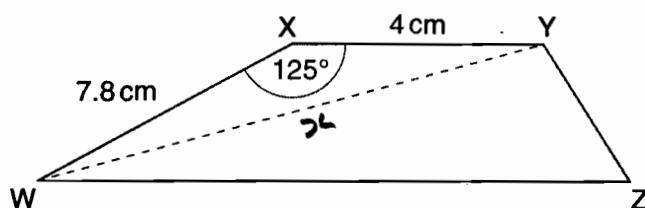
$$\sin 34^\circ = \frac{h}{5.2}$$

$$5.2 \sin 34^\circ = h$$

$$h = 2.91 \text{ to 3 s.f.}$$

(b) 2.91 cm [3]

(c) WXYZ is a trapezium.



NOT TO SCALE

Calculate WY. Let  $WY = x$

$$\text{Cosine Rule } x^2 = 7.8^2 + 4^2 - 2 \times 7.8 \times 4 \times \cos 125^\circ$$

$$x^2 = 112.631$$

$$x = \sqrt{112.631} = 10.6 \text{ cm}$$

(c) 10.6 cm [3]

12 Simplify.

$$(a) t^2 \times t^7 = t^{2+7} = t^9$$

$$(a) \underline{t^9} \quad [1]$$

$$(b) \frac{s^3}{s^6} = s^{3-6} = s^{-3}$$

$$(b) \underline{s^{-3} \text{ or } \frac{1}{s^3}} \quad [1]$$

$$(c) s^3 t^3 \times s^4 t^2 = s^{3+4} t^{3+2} = s^7 t^5$$

$$(c) \underline{s^7 t^5} \quad [2]$$

$$(d) (s^3 t^4)^2 = s^{3 \times 2} t^{4 \times 2} = s^6 t^8$$

$$(d) \underline{s^6 t^8} \quad [2]$$

13  $y$  is proportional to the square of  $x$ .  
 $y = 18$  when  $x = 6$ .

$$y = kx^2$$

(a) Find an equation connecting  $y$  and  $x$ .

$$\text{Subst } y = 18, x = 6$$

$$18 = k \times 6^2$$

$$18 = 36k$$

$$\frac{18}{36} = k$$

$$k = \frac{1}{2}$$

$$y = \frac{1}{2} x^2$$

$$(a) \underline{y = \frac{1}{2} x^2} \quad [3]$$

(b) Find the values of  $x$  when  $y = 5$ .

$$\text{Subst } y = 5$$

$$5 = \frac{1}{2} x^2$$

$$10 = x^2$$

$$x = \pm \sqrt{10}$$

$$(b) \underline{\pm 3.16} \quad [2]$$

- 14 A toy car travels 180 cm, correct to the nearest 10 cm.  
It takes 7 seconds, correct to the nearest second, to travel this distance.

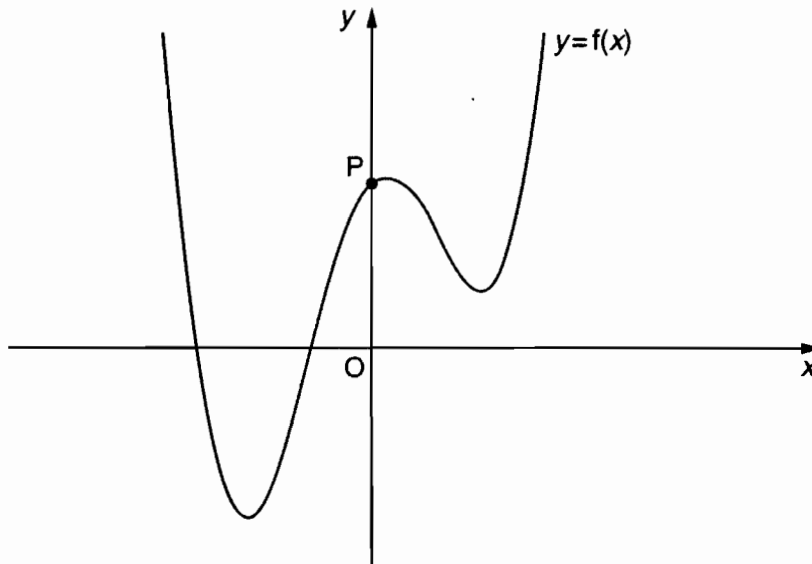
Work out the greatest possible value of the average speed of the toy car.  
You must show all your working.

greatest speed from  $\frac{\text{greatest distance}}{\text{smallest time}}$

$$= \frac{185 \text{ cm}}{6.5 \text{ s}} = 28.46$$

28.46 cm/s [4]

- 15 The diagram shows the graph of  $y = f(x)$ .



The graph passes through the point P (0, 2).

Write down the coordinates of the image of P when  $y = f(x)$  is transformed to

(a)  $y = f(x) - 3$ , Drops down by 3 units

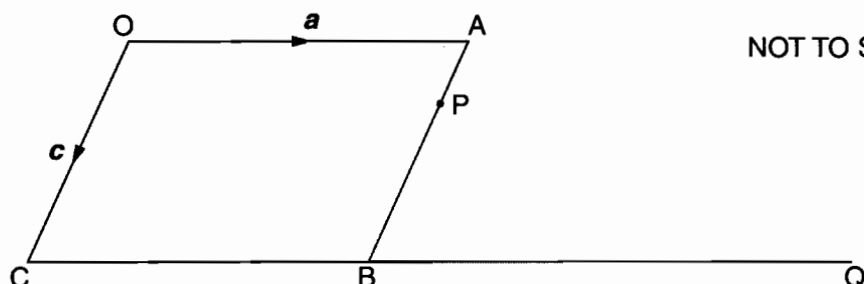
(a) ( 0 , -1 ) [1]

(b)  $y = f(x - 3)$ , Moves right 3 units

(b) ( 3 , 2 ) [1]

16 OABC is a parallelogram.

$$\vec{OA} = \underline{a} \quad \vec{OC} = \underline{c}$$



P is the point on AB such that  $\vec{AP} = \frac{1}{4} \vec{AB}$ .

CBQ is a straight line such that  $CB : BQ = 1 : 3$ .

(a) Write down, in terms of  $\underline{a}$  and  $\underline{c}$ , the vectors

$$(i) \vec{AP} = \frac{1}{4} \vec{AB} = \frac{1}{4} \vec{OC} = \frac{1}{4} \underline{c}$$

$$(a)(i) \underline{\frac{1}{4} c} \quad [1]$$

$$(ii) \vec{OP} = \vec{OA} + \vec{AP} \\ = \underline{a} + \frac{1}{4} \underline{c}$$

$$(ii) \underline{a + \frac{1}{4} c} \quad [1]$$

$$(iii) \vec{BQ} = 3\vec{CB} = 3\vec{OA} \\ = 3\underline{a}$$

$$(iii) \underline{3a} \quad [1]$$

$$(iv) \vec{OQ} = \vec{OC} + \vec{CB} + \vec{BQ} \\ = \underline{c} + \underline{a} + 3\underline{a}$$

$$(iv) \underline{c + 4a} \quad [1]$$

(b) Explain, using vectors, why O, P and Q lie on a straight line.

$$\vec{OP} = \underline{a} + \frac{1}{4} \underline{c}$$

$$\vec{OQ} = 4\underline{a} + \underline{c} = 4 \vec{OP}$$

$\therefore \vec{OQ}$  is parallel to  $\vec{OP}$ , both passing through O

$\therefore OPQ$  is a straight line. (b) \_\_\_\_\_ [1]