

A* 76, A 60, B 44, C 28,



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**GENERAL CERTIFICATE OF SECONDARY EDUCATION
MATHEMATICS SYLLABUS A**

J512/04

Paper 4 (Higher Tier)

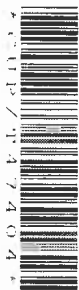
MONDAY 2 JUNE 2008

Afternoon
Time: 2 hours

Candidates answer on the question paper
Additional materials (enclosed): None

Additional materials (required):
Electronic calculator
Geometrical instruments
Tracing paper (optional)

Solutions



Candidate
Forename

Candidate
Surname

Centre
Number

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Candidate
Number

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INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use blue or black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- **Do not** write in the bar codes.
- Write your answer to each question in the space provided.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **100**.
- You are expected to use an electronic calculator for this paper.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.

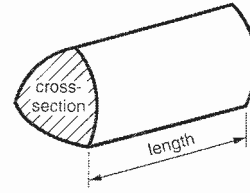
FOR EXAMINER'S USE

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This document consists of **20** printed pages.

Formulae Sheet: Higher Tier

Volume of prism = (area of cross-section) \times length

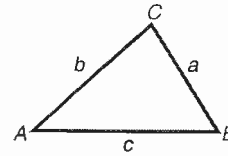


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

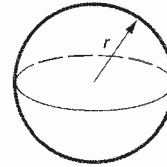
Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



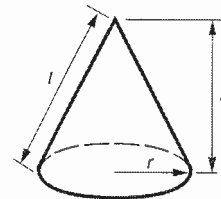
Volume of sphere = $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3} \pi r^2 h$

Curved surface area of cone = $\pi r l$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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1 (a) Calculate.

$$(i) \frac{7.6}{12.5 - 6.8} = \frac{7.6}{(12.5 - 6.8)} = 1.3\dot{3}$$

(a)(i) 1.3\dot{3} [1]

$$(ii) \left(\frac{5}{0.4}\right)^2$$

(ii) 156.25 [2]

(b) (i) Calculate.

$$\sqrt{25.3 - 2.7^3} = \sqrt{(25.3 - 2.7^3)}$$

Write down all the figures on your calculator display.

(b)(i) 2.370021097 [1]

(ii) Give your answer to part (b)(i) correct to 2 significant figures.

(ii) 2.4 [1]

2 A formula for changing degrees Fahrenheit (F) into degrees Celsius (C) is

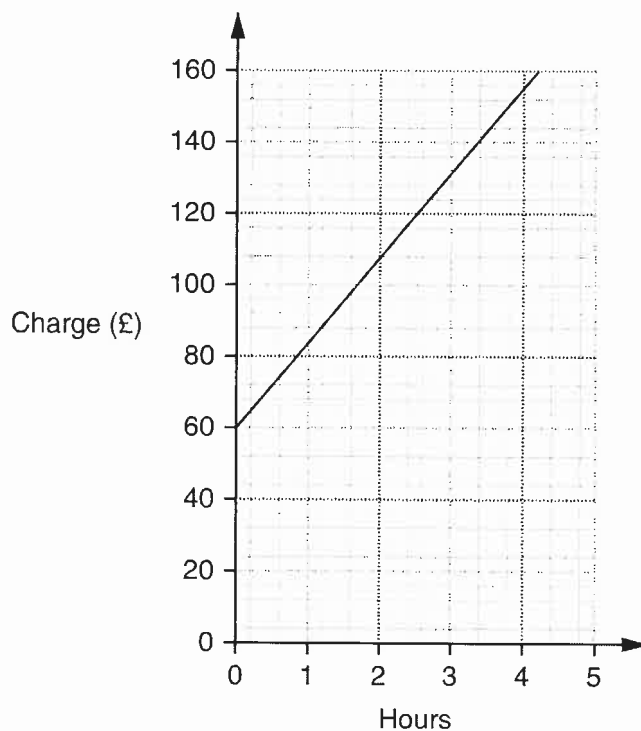
$$C = (F - 32) \times \frac{5}{9}$$

Calculate the temperature in degrees Celsius for a temperature of 46 degrees Fahrenheit.
Give your answer to a suitable degree of accuracy.

$$C = (46 - 32) \times \frac{5}{9} = 14 \times \frac{5}{9} = 7.7\dot{7}$$

7.8 °C [3]

- 3 An electrician uses this graph to work out how much to charge for each job.



The charge for a job is made up of a fixed fee plus an amount for the time that the job lasts.

- (a) How much is the fixed fee?

(a) £ 60 [1]

- (b) How much would the electrician charge for a job that lasts 5 hours?

From graph variable part of fee = £24 per hour
 5 hour job is £60 + 5 × £24 (b) £ 180 [1]

- (c) The charge, £C, can be written as a formula in terms of the fixed fee, the rate per hour and the number of hours, h , for the job.

Complete the formula.

.....

 $C = \underline{60} + \underline{24} \times h$ [2]

4 Clare is reading a poetry book.

- (a) The shortest poem in the book has one verse and the longest poem has six verses. The poems in the book are from ten to fifteen lines in length.

Design a two-way table for Clare to record the number of verses and the number of lines for the poems in the book.

[3]

		Verses						
		1	2	3	4	5	6	TOTALS
Lines	10							
	11							
	12							
	13							
	14							
	15							
TOTAL								

- (b) Clare counted the number of letters in each word of one poem. There were 28 words in the poem.

She drew this table to show her results.

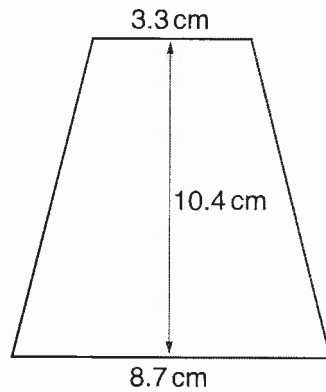
Number of letters	Frequency	Num x FREQ
1	2	2
2	5	10
3	6	18
4	8	32
5	2	10
6	3	18
7	2	14
Total	28	104

Calculate the mean number of letters per word in the poem.

$$\text{Mean} = \frac{104}{28} = 3.714$$

(b) 3.7 to 2 s.f. [3]

- 5 Calculate the area of this trapezium.



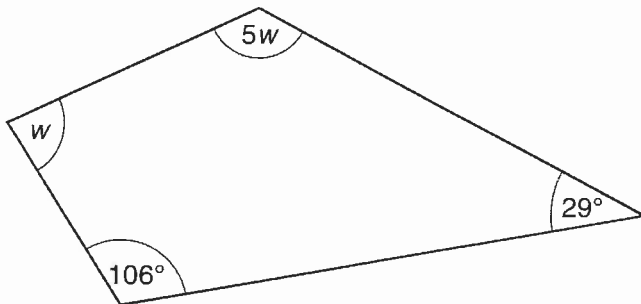
NOT TO SCALE

$$\text{Area} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} (8.7 + 3.3) \times 10.4$$

$$= 62.4 \text{ cm}^2 [2]$$

- 6 (a) A quadrilateral has angles as shown.



NOT TO SCALE

Work out the value of w .

Angles add up to 360°

$$5w + w + 106 + 29 = 360$$

$$6w = 360 - 106 - 29$$

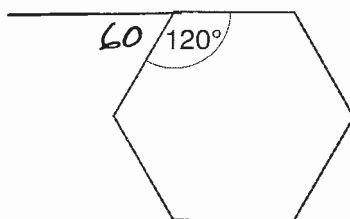
$$6w = 225$$

$$w = \frac{225}{6}$$

$$w = 37.5$$

(a) $37.5^\circ [4]$

- (b) (i) Here is a regular hexagon.



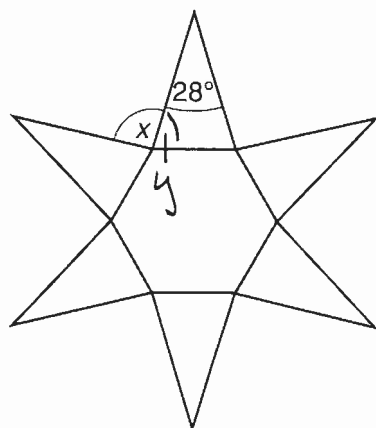
Explain why each interior angle is 120°.

$$\text{Exterior angle} = \frac{360}{6} = 60^\circ$$

$$\begin{aligned} \text{Interior angle} &= 180 - \text{exterior angle} \\ &= 180 - 60 = 120^\circ \end{aligned}$$

[2]

- (ii) This shape is made from six congruent isosceles triangles and a regular hexagon.



NOT TO SCALE

Find the size of angle x.

$$\text{Base angle of isos } \Delta, y = \frac{180 - 28}{2} = 76^\circ$$

$$\text{Angles at a point add up to } 360^\circ$$

$$\therefore 76 + 76 + 120 + x = 360$$

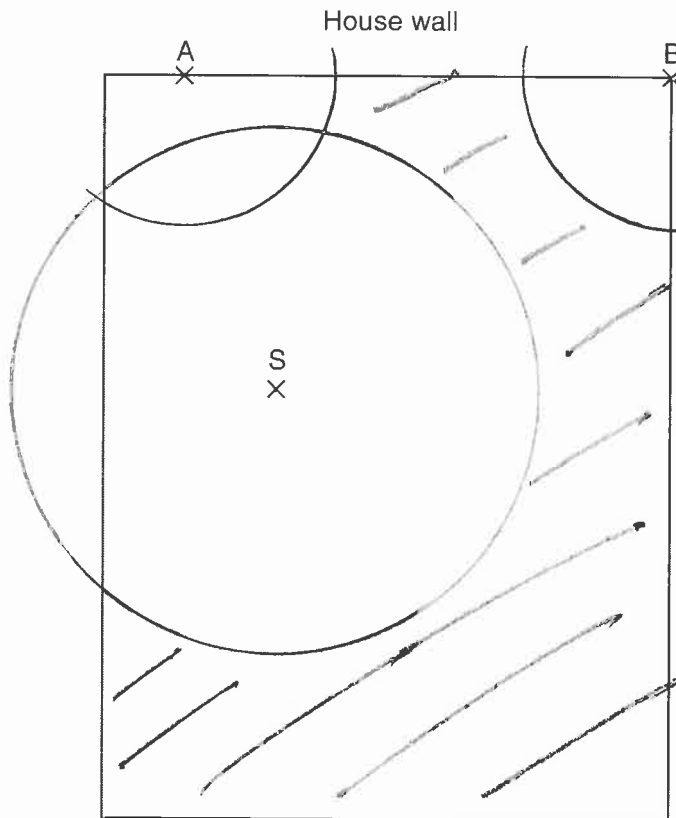
$$272 + x = 360$$

$$x = 360 - 272$$

$$x = 88^\circ$$

$$(b)(ii) \quad 88^\circ \quad [3]$$

- 7 The diagram shows the garden of a house.
There is a security light, S, in the garden and two security lights, A and B, on the house wall.
The lights are at ground level.



Scale: 1 cm represents 2 m

*Fox can move
in shade area
outside the circles*

The security light in the garden comes on when it detects movement within 7 m.
Each security light on the house wall comes on when it detects movement within 4 m.

A fox is in the garden.

Indicate clearly the region where the fox can move **without** making any of the security lights come on.

[3]

- 8 Use trial and improvement to solve this equation.

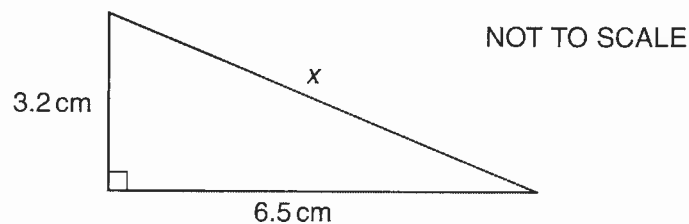
$$x^3 + x = 8$$

Give your answer to one decimal place.
Show all your trials and their outcomes.

x	$x^3 + x$	
1	$1^3 + 1 = 2$	too low
2	$2^3 + 2 = 10$	too high
1.5	$1.5^3 + 1.5 = 4.875$	too low
1.8	$1.8^3 + 1.8 = 7.632$	too low
1.9	$1.9^3 + 1.9 = 8.759$	too high
1.85	$1.85^3 + 1.85 = 8.18$	too high
so $x = 1.8$ to 1 d.p		

$$x = 1.8 \quad [4]$$

- 9 Calculate the value of x .



Pythagoras $x^2 = 3.2^2 + 6.5^2$

$$x^2 = 52.49$$

$$x = \sqrt{52.49}$$

$$x = 7.244998$$

$$x = 7.24 \text{ cm} \quad [3]$$

10 (a) (i) Solve.

$$4x - 5 < 9$$

$$4x < 9 + 5$$

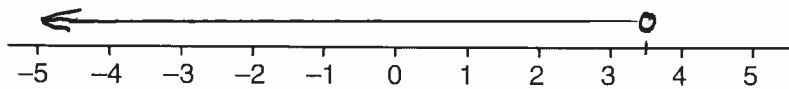
$$4x < 14$$

$$x < \frac{14}{4}$$

$$\frac{7}{2}$$

$$(a)(i) \quad x < 3.5 \quad [2]$$

(ii) Represent your solution to part (a)(i) on the number line below.



[1]

(b) Expand and simplify.

$$(i) \quad 2(4x + 5) - 3(x - 2)$$

$$= 8x + 10 - 3x + 6$$

$$= 5x + 16$$

$$(b)(i) \quad 5x + 16 \quad [3]$$

$$(ii) \quad (x + 7)(x - 3)$$

$$= x^2 + 7x - 3x - 21$$

$$= x^2 + 4x - 21$$

$$(ii) \quad x^2 + 4x - 21 \quad [2]$$

- 11 (a) Lewis was baking bread.
He made the dough and left it to rise.
The volume of the dough before it had risen was 680 cm^3 .
The volume of the dough after it had risen was 1258 cm^3 .

Calculate the percentage increase in the volume of the dough.

$$\frac{1258}{680} = 1.85 \quad \text{An } 85\% \text{ increase}$$

(a) 85 % [3]

- (b) Debbie made ice-lollies from fruit juice.
When the fruit juice was frozen its volume increased by 12%.
The volume of the frozen ice-lollies was 840 cm^3 .

Calculate the volume of the fruit juice before it was frozen.

$$840 \text{ cm}^3 = 112\% \text{ of original}$$

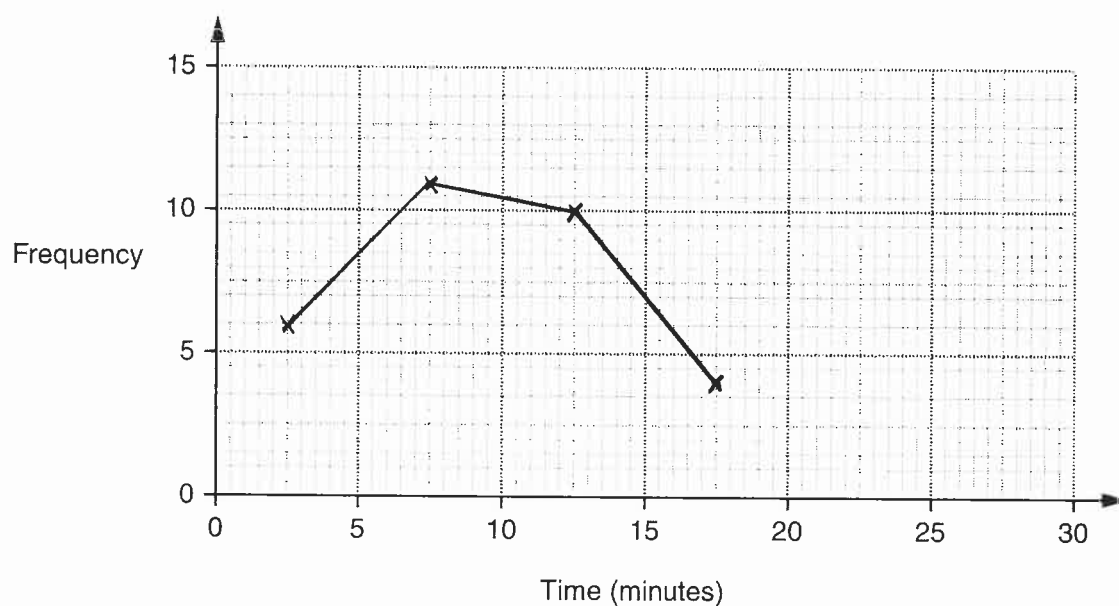
$$840 \div 1.12 = 750 \text{ cm}^3$$

(b) 750 cm^3 [3]

- 12 Andrew kept a daily record of the time in minutes he used his phone during two months. The table summarises the data for July.

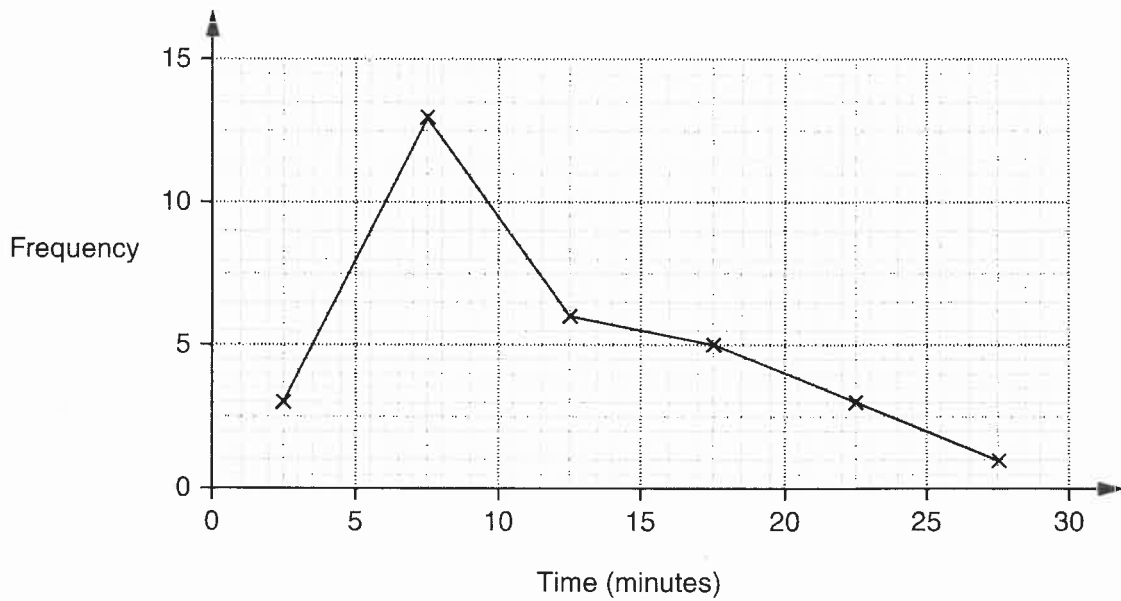
Time (m minutes)	Frequency
$0 < m \leq 5$	6
$5 < m \leq 10$	11
$10 < m \leq 15$	10
$15 < m \leq 20$	4

- (a) Draw a frequency polygon for these data.



[2]

(b) The frequency polygon below represents the data Andrew collected for August.



Write down one similarity and one difference between the daily times for which Andrew used his phone during July and August.

Similarity: Highest frequency for both months was $5 < m \leq 10$

Difference: Made calls over 20 mins in August but not in July [2]

- 13 (a) What is the lowest common multiple (LCM) of 72 and 42?

72, 144, 216, 288, 360, 432, 504
 42, 84, 126, 168, 210, 252, 294, 336, 378, 420, 462, 504

(a) 504 [2]

- (b) What is the highest common factor (HCF) of 98 and 42?

$$\begin{array}{r} 2 \overline{)98} \\ 7 \overline{)49} \\ 7 \overline{)7} \\ 1 \end{array} \quad \begin{array}{r} 2 \overline{)42} \\ 3 \overline{)21} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$\begin{aligned} 98 &= 2 \times 7 \times 7 \\ 42 &= 2 \times 3 \times 7 \\ \text{HCF} &= 2 \times 7 \\ &= 14 \end{aligned}$$

(b) 14 [2]

- (c) Calculate the following.
 Give your answers in standard form.

(i) $(6.4 \times 10^{20}) + (5 \times 10^{19})$

$$6.4 \text{ Exp } 20 + 5 \text{ Exp } 19 = 6.9 \text{ Exp } 20$$

(c)(i) 6.9×10^{20} [1]

(ii) $(2.7 \times 10^{-5}) \times (7.8 \times 10^2)$

$$\begin{aligned} 2.7 \text{ Exp } -5 \times 7.8 \text{ Exp } 2 &= 0.02106 \\ &= 2.106 \times 10^{-2} \end{aligned}$$

(ii) 2.106×10^{-2} [2]

A better method for 13 a)

$$\begin{array}{r} 2 \overline{)72} \\ 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \overline{)3} \\ 1 \end{array} \quad \begin{array}{r} 2 \overline{)42} \\ 3 \overline{)21} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$42 = 2 \times 3 \times 7$$

LCM must contain all
 factors of both numbers

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 2 \times 3 \times 3 \times 7 \\ &= 504 \end{aligned}$$

14 Solve, algebraically, these simultaneous equations.

$$\begin{array}{l} 2x - 3y = 14 \quad (1) \\ 6x + 4y = 3 \quad (2) \end{array}$$

$$(1) \times 4 \quad 8x - 12y = 56 \quad (3)$$

$$(2) \times 3 \quad 18x + 12y = 9 \quad (4)$$

$$(3) + (4) \quad 26x = 65$$

$$x = \frac{65}{26}$$

$$x = 2.5$$

Subst for x in (1)

$$2(2.5) - 3y = 14$$

$$5 - 3y = 14$$

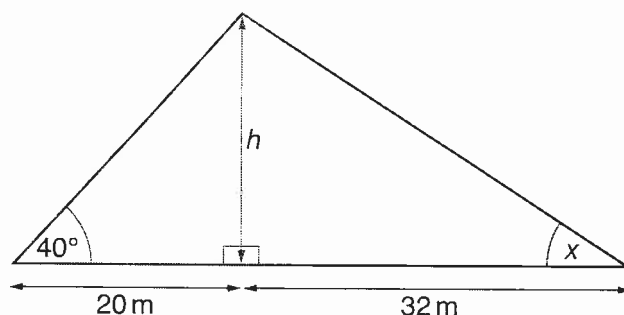
$$-3y = 14 - 5$$

$$-3y = 9$$

$$y = \frac{9}{-3} = -3$$

$$x = 2.5 \quad y = -3 \quad [3]$$

15



NOT TO SCALE

(a) Calculate h .

$$\tan 40^\circ = \frac{\text{opp}}{\text{adj}} = \frac{h}{20}$$

$$20 \tan 40^\circ = h$$

$$16.78 \text{ m} = h$$

$$(a) \quad 16.78 \text{ m} \quad [3]$$

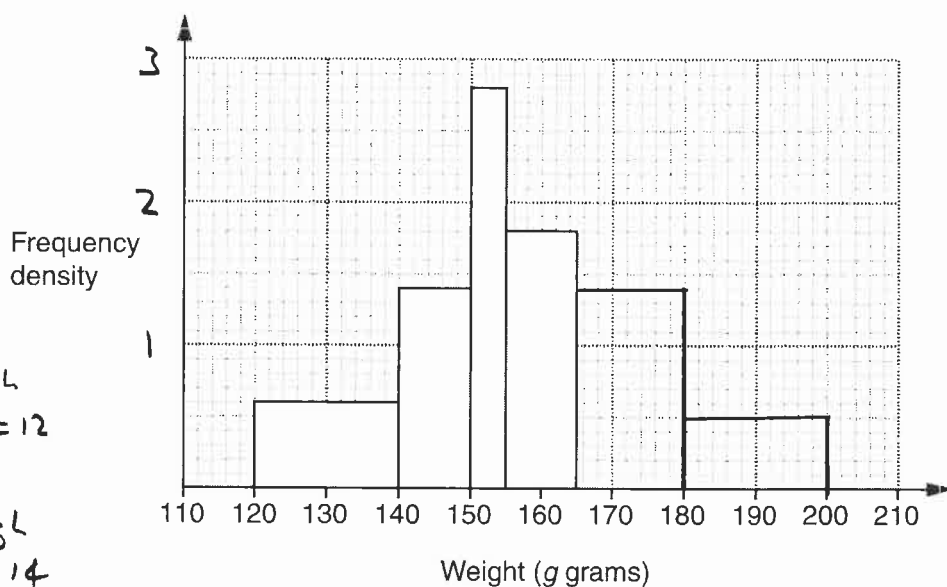
(b) Calculate angle x .

$$\tan x = \frac{h}{32} = \frac{16.78199}{32}$$

$$x = \tan^{-1} \left(\frac{16.78199}{32} \right) = 27.67^\circ$$

$$(b) \quad 27.7^\circ \quad [3]$$

16 William is drawing a histogram to show information about the weights of some pears.



First bar
must be 0.6 high
since $0.6 \times 20 = 12$
2nd bar
must be 1.4 high
since $1.4 \times 10 = 14$

(a) Complete the frequency table.

Weight (g grams)	Frequency
$120 < g \leq 140$	12
$140 < g \leq 150$	14
$150 < g \leq 155$	14
$155 < g \leq 165$	18
$165 < g \leq 180$	21
$180 < g \leq 200$	10

$$2.8 \times 5 = 14$$

$$1.8 \times 10 = 18$$

$$21 \div 15 = 1.4 \text{ high}$$

$$10 \div 20 = 0.5 \text{ high}$$

[2]

(b) Complete the histogram, including a scale.

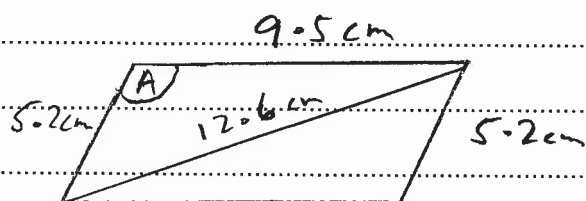
Working

$$\text{Freq} \div \text{group width} = \text{Freq density}$$

[3]

- 17 A parallelogram has sides 5.2 cm and 9.5 cm.
The longer diagonal is 12.6 cm.

Calculate the size of an obtuse angle of the parallelogram.



Find angle A

$$\text{Cosine rule } a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc \cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{5.2^2 + 9.5^2 - 12.6^2}{2 \times 5.2 \times 9.5} = -0.4197 \quad \underline{114.8}^\circ [3]$$

$$A = \cos^{-1}(-0.4197) = 114.8^\circ$$

- 18 (a) Express 8^p as a power of 2.

$$8^p = (2^3)^p = 2^{3 \times p} = 2^{3p}$$

(a) $\underline{2^{3p}} [2]$

- (b) Find the exact value of p where

$$2^4 \times 8^p = 2^6$$

$$2^4 \times 2^{3p} = 2^6$$

$$2^{4+3p} = 2^6$$

$$\Rightarrow 4 + 3p = 6$$

$$3p = 6 - 4$$

$$3p = 2$$

$$p = \frac{2}{3}$$

(b) $\underline{p = \frac{2}{3}} [2]$

19 (a) Solve.

$$x^2 - 8x + 11 = 0$$

Give your answers correct to 2 decimal places.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{64 - 4 \times 1 \times 11}}{2} = \frac{8 \pm \sqrt{20}}{2}$$

$$x = 6.24 \text{ or } x = 1.76$$

$$(a) \underline{x = 6.24 \text{ or } 1.76} [3]$$

(b) Find a and b when

$$x^2 + 6x - 14 = (x + a)^2 + b.$$

$$= (x + 3)^2 - 14 - 9$$

$$= (x + 3)^2 - 23$$

$$(b) \underline{a = 3} \quad \underline{b = -23} [3]$$

20 Simplify.

$$\frac{x^2 + 7x}{x^2 - 49} = \frac{x \cancel{(x+7)}}{\cancel{(x+7)}(x-7)} = \frac{x}{x-7}$$

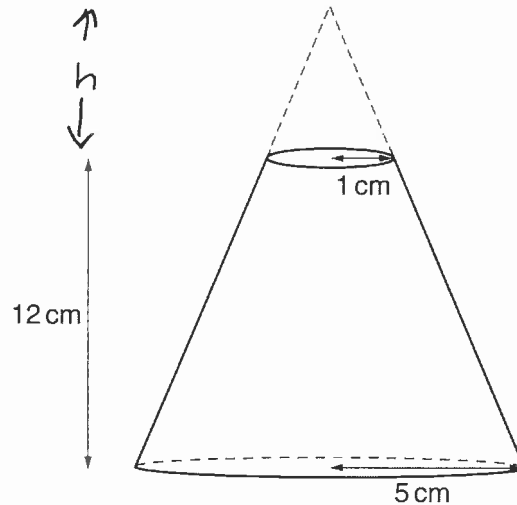
$$\frac{x}{x-7}$$

[3]

TURN OVER FOR QUESTION 21

- 21 The diagram shows a frustum of a cone.

It is made by removing a cone of radius 1 cm from a cone of radius 5 cm.
The height of the frustum is 12 cm.



Calculate the volume of the frustum.

$$\text{Length ratio small cone : large cone} = 1 : 5$$

$$\therefore h : h + 12 = 1 : 5$$

$$\Rightarrow h + 12 = 5h$$

$$12 = 5h - h = 4h$$

$$\frac{12}{4} = h$$

$$\text{so } h = 3 \text{ cm}$$

Volume of frustum = Volume of large cone - Vol of small cone

$$\text{Vol cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 5^2 \times 15 - \frac{1}{3}\pi \times 1^2 \times 3$$

$$= 125\pi - \pi$$

$$= 124\pi = 389.56 \text{ cm}^3$$

$$\underline{389.6} \text{ cm}^3 [6]$$