

A*87 A72 B56 C41



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**GENERAL CERTIFICATE OF SECONDARY EDUCATION
MATHEMATICS SYLLABUS A**

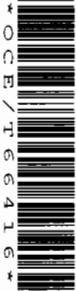
J512/03

Paper 3
(Higher Tier)

Solutions

**Monday 18 May 2009
Afternoon**

Duration: 2 hours



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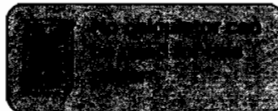
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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.

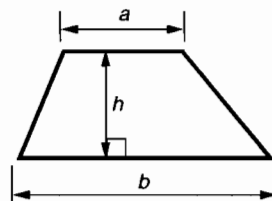
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **100**.
- This document consists of **20** pages. Any blank pages are indicated.

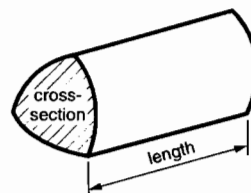


Formulae Sheet: Higher Tier

Area of trapezium = $\frac{1}{2}(a + b)h$



Volume of prism = (area of cross-section) x length

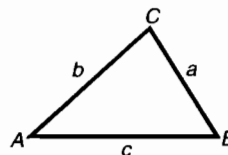


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

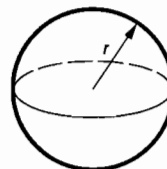
Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$



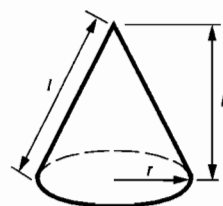
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

- 1 Mr Smith did a survey of how students travelled to school. He displayed his results in a table.

Complete the table.

	Bus	Walk	Car	Total
Boys	21	23	13	57
Girls	19	8	16	43
Total	40	31	29	100

[3]

- 2 A jacket can be bought in a shop or online.

<p><u>Shop price</u></p> <p>Jacket, usually £75</p> <p>Sale, $\frac{1}{5}$ off</p>

<p><u>Online price</u></p> <p>Jacket £50</p> <p>plus 15% postage and packing</p>
--

Which method of buying the jacket is cheaper, and by how much?
Show all your working clearly.

$$\begin{array}{r}
 15 \\
 \hline
 5 \overline{) 75} \\
 \hline
 \end{array}$$

£15 off

$$£75 - £15 = £60$$

$$10\% \text{ of } £50 = £5$$

$$5\% \text{ of } £50 = £2.50$$

$$15\% \text{ of } £50 = £7.50$$

$$\text{Total } £50 + £7.50 = £57.50$$

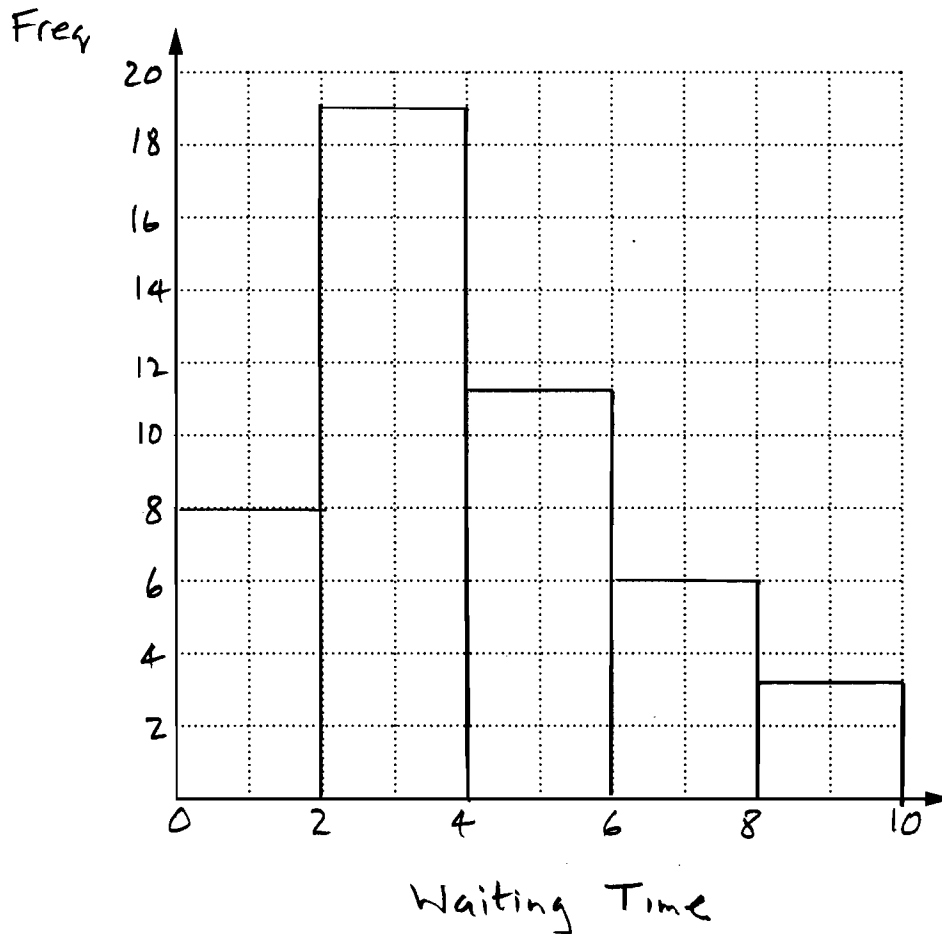
$$\text{Online cheaper by } £60 - £57.50 = £2.50$$

online is cheaper by £ 2.50 [5]

- 3 (a) The table shows the distribution of waiting times (in minutes) that customers spent at the checkout of a supermarket.

Waiting time (minutes)	Frequency
0 up to 2	8
2 up to 4	19
4 up to 6	11
6 up to 8	6
8 up to 10	3

- (i) Draw a grouped frequency diagram to show this information. Show your scales and label your axes clearly.



[3]

(ii) Write down the modal class for these waiting times.

(a)(ii) 2-4 minutes [1]

(iii) One of these customers is chosen at random.

What is the probability that this customer waited 6 minutes or more?

MORE THAN 6 MINUTES → 9 people

TOTAL PEOPLE → 47

(iii) $\frac{9}{47}$ [2]

(b) At the supermarket, Jack is doing a survey about eating sweets. This is his questionnaire.

'How many chocolate bars do you eat?'

A few A lot

(Please tick one box.)

Write down two things that are wrong with Jack's questionnaire.

- 1 No time period specified
- 2 A few and a lot are not defined, they need to be specific [2]
3. Too few options, cannot say 0 for example

- 4 (a) Show that $x = 2$ is the solution of this equation.

$$9x - 1 = 4x + 9$$

$$9x - 4x = 9 + 1$$

$$5x = 10$$

$$x = \frac{10}{5}$$

$$x = 2$$

[2]

- (b) Solve.

$$\frac{x}{2} - 3 = 5$$

$$x - 6 = 10$$

$$x = 10 + 6$$

$$x = 16$$

(b) $x = 16$ [2]

- 5 (a) In a carton of *Squashy*, orange juice and water are mixed in the ratio 3 : 7.

How many litres of orange juice are needed to make 60 litres of *Squashy*?

$$\text{Parts} = 3 + 7 = 10$$

$$10 \text{ parts} = 60 \text{ litres}$$

$$1 \text{ part} = 6 \text{ litres}$$

$$\text{Orange} = 3 \times 6 = 18 \text{ litres}$$

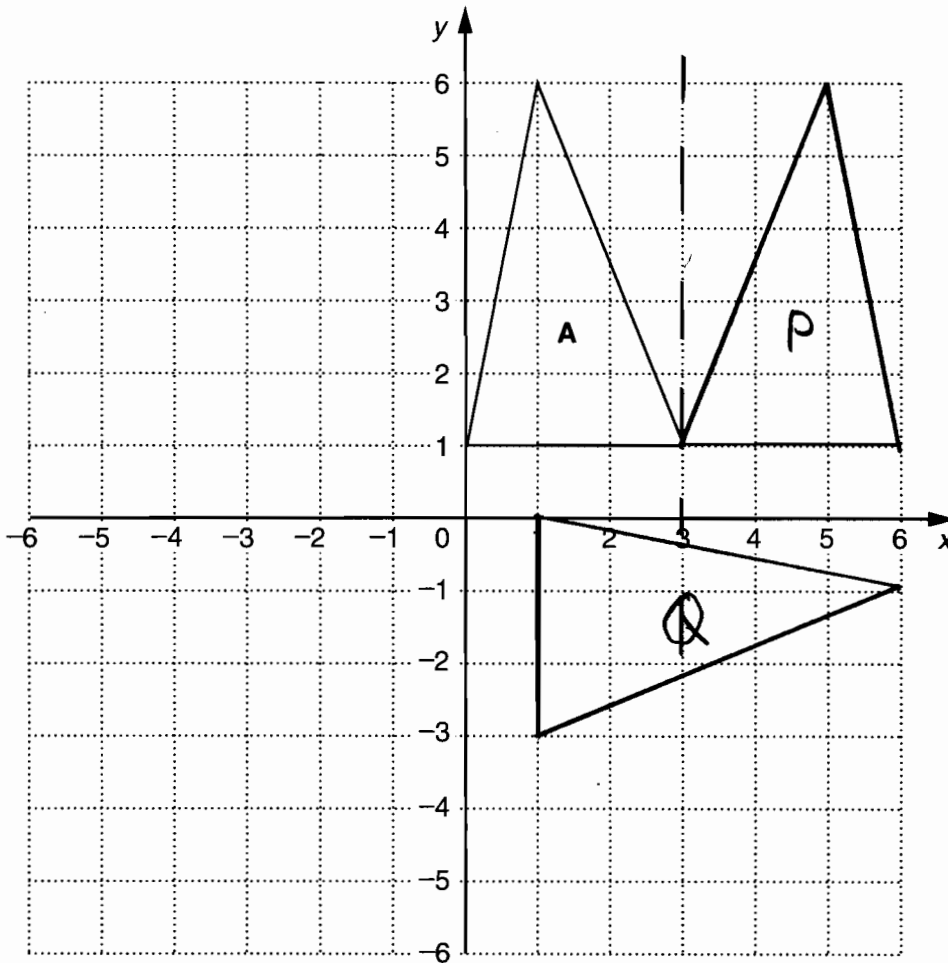
(a) 18 litres [2]

- (b) One carton contains 150 ml of *Squashy*, correct to the nearest millilitre.

What is the least possible amount of *Squashy* that could be contained in the carton?

(b) 149.5 ml [1]

6



- (a) Triangle A is drawn on a 1 cm square grid.

Work out the area of triangle A.

$$\frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 3 \times 5 = \frac{15}{2} = 7.5 \text{ cm}^2$$

(a) 7.5 cm² [2]

- (b) Reflect triangle A in the line $x = 3$.
Label the image P.

[2]

- (c) Rotate triangle A 90° clockwise about (0,0).
Label the image Q.

[3]

- 7 (a) List the integer values, n , which satisfy

$$3 < n \leq 7.$$

4, 5, 6, 7

(a) 4, 5, 6, 7 [2]

- (b) (i) Solve this inequality.

$$3x - 2 > 4$$

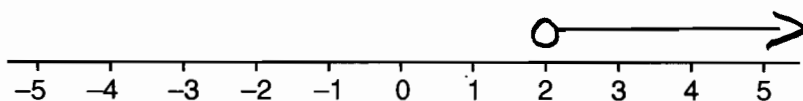
$$3x > 4 + 2$$

$$3x > 6$$

$$x > \frac{6}{3}$$

(b)(i) $x > 2$ [2]

- (ii) Show your solution to part (i) on the number line below.



[1]

- 8 (a) Here are the first four terms of a sequence.

25 20 15 10

Find an expression for the n th term of this sequence.

going down in 5's so $-5n$ in formula

For first term, some number $-5 \times 1 = 25$

$30 - 5 = 25$ so

(a) $n^{\text{th}} \text{ term} = 30 - 5n$ [2]

- (b) Here are the first four terms of another sequence.

1 4 9 16

The n th term of this sequence is n^2 .

Write down an expression for the n th term of the following sequence.

3 6 11 18

Each term is two more so

(b) $n^2 + 2$ [1]

- 9 As a product of prime factors,

$$24 = 2 \times 2 \times 2 \times 3.$$

- (a) Write 40 as a product of prime factors.

$$\begin{array}{r} 2 \overline{)40} \\ 2 \overline{)20} \\ 2 \overline{)10} \\ 5 \overline{)5} \\ 1 \end{array}$$

$40 = 2 \times 2 \times 2 \times 5$

(a) $40 = 2 \times 2 \times 2 \times 5$ [2]

- (b) (i) Work out the highest common factor (HCF) of 24 and 40.

$2 \times 2 \times 2$ is HCF

$= 8$

(b)(i) 8 [2]

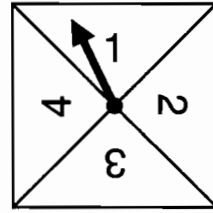
- (ii) Work out the least common multiple (LCM) of 24 and 40.

LCM has factors of both

$2 \times 2 \times 2 \times 3 \times 5 = 120$

(ii) 120 [2]

- 10 Jo wanted to know if this spinner was fair. She spun it a number of times.



The table shows her results.

Number	1	2	3	4
Frequency	115	129	132	124

$$\begin{array}{r}
 115 \\
 129 \\
 132 \\
 124 + \\
 \hline
 500
 \end{array}$$

- (a) What is the relative frequency of obtaining a 2?

Total spins = $115 + 129 + 132 + 124 = 500$

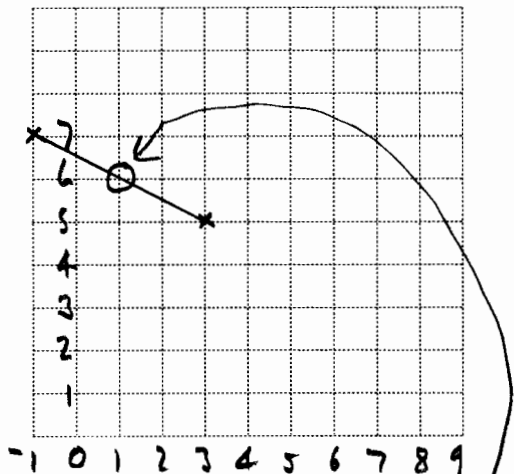
2 occurred 129 times

(a) $\frac{129}{500}$ [2]

- (b) Do the results suggest that the spinner is fair? Give a reason for your answer.

YES because the frequencies are all close to the expected values of $\frac{500}{4} = 125$ [1]

- 11 Work out the coordinates of the midpoint of the line joining the points (3,5) and (-1,7). You may use the grid to help you.



Graph solution
(1, 6)

2nd Method

Midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$= \left(\frac{3 + (-1)}{2}, \frac{5 + 7}{2} \right)$

$= \left(\frac{2}{2}, \frac{12}{2} \right)$

$= (1, 6)$ [2]

12 (a) Multiply out.

$$5(3x - 4) = 15x - 20$$

(a) 15x - 20 [2]

(b) Factorise.

$$\frac{2a^2 + 8ab}{2a^2 + 8ab} = 2a(a + 4b)$$

(b) 2a(a + 4b) [2]

(c) (i) Write down the value of 3^0 .

(c)(i) $3^0 = 1$ [1]

(ii) Simplify.

$$\frac{8x^6y^5}{2x^4y} = 4x^2y^4$$

(ii) $4x^2y^4$ [3]

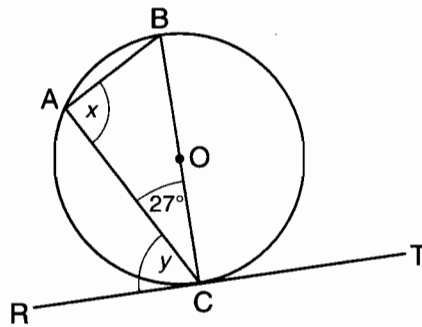
(iii) Simplify.

$$(7^3)^5 = 7^{3 \times 5} = 7^{15}$$

Give your answer as a power of 7.

(iii) 7^{15} [1]

- 13 (a) A, B and C are points on the circle, centre O.
 RCT is a tangent to the circle.
 BOC is a straight line.



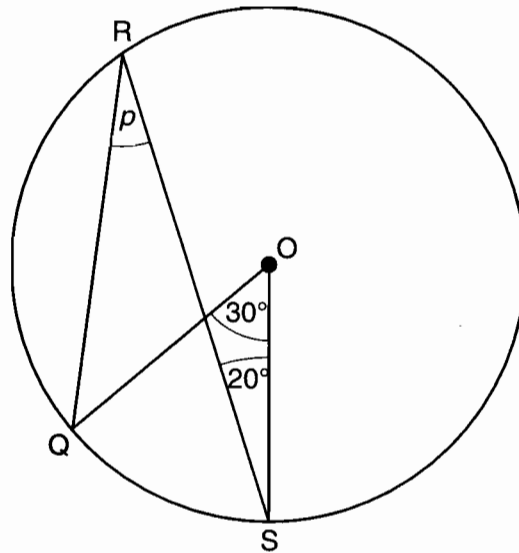
NOT TO SCALE

Complete each of these statements by giving a geometrical reason.

(i) Angle $x = 90^\circ$ because angle in a semi-circle
 _____ [1]

(ii) Angle $y = 63^\circ$ because Angle between radius and tangent = 90° . So $90 - 27 = 63^\circ$
 _____ [1]

- (b) (i) Q, R and S are points on a circle, centre O.



NOT TO SCALE

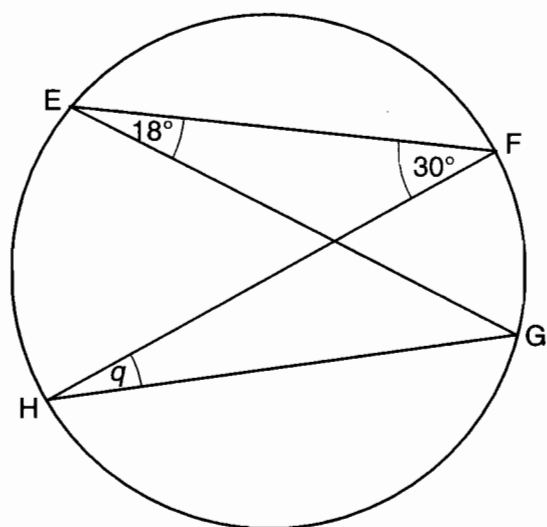
Work out the size of angle p .

$P = 15^\circ$ (angle at centre is twice angle at circumference)

(b)(i) _____ $^\circ$ [1]

$P = 15^\circ$

(ii) E, F, G and H are points on a circle.



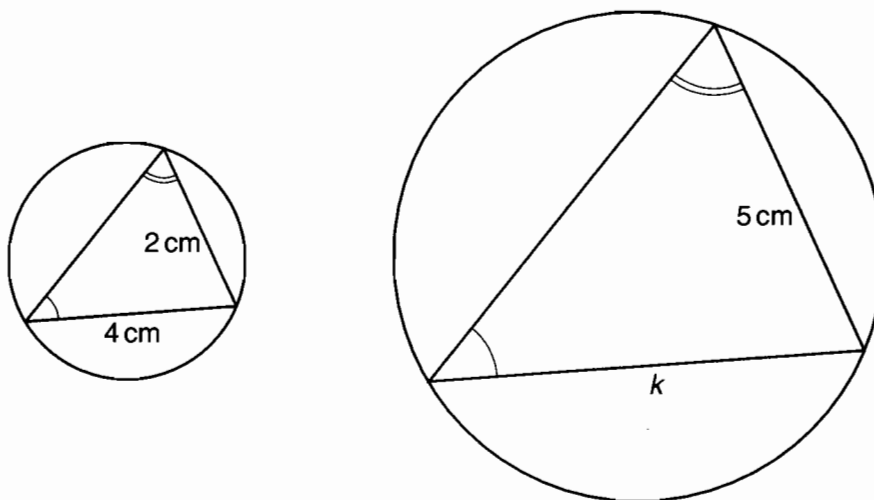
NOT TO
SCALE

$$q = 18^\circ \text{ (angles in same segment are equal)}$$

What is the size of angle q ?

(ii) $q = 18^\circ$ ° [1]

(c) These two triangles are similar.



NOT TO
SCALE

Work out the length k .

$$\text{Ratio large : small} = 5 : 2$$

$$k = \frac{4 \times 5}{2} = \frac{20}{2} = 10 \text{ cm}$$

(c) 10 cm cm [2]

14 Work out.

$$2\frac{1}{2} \times 1\frac{2}{3}$$

Give your answer as a mixed number.

$$= \frac{5}{2} \times \frac{5}{3}$$

$$= \frac{25}{6}$$

$$= 4\frac{1}{6}$$

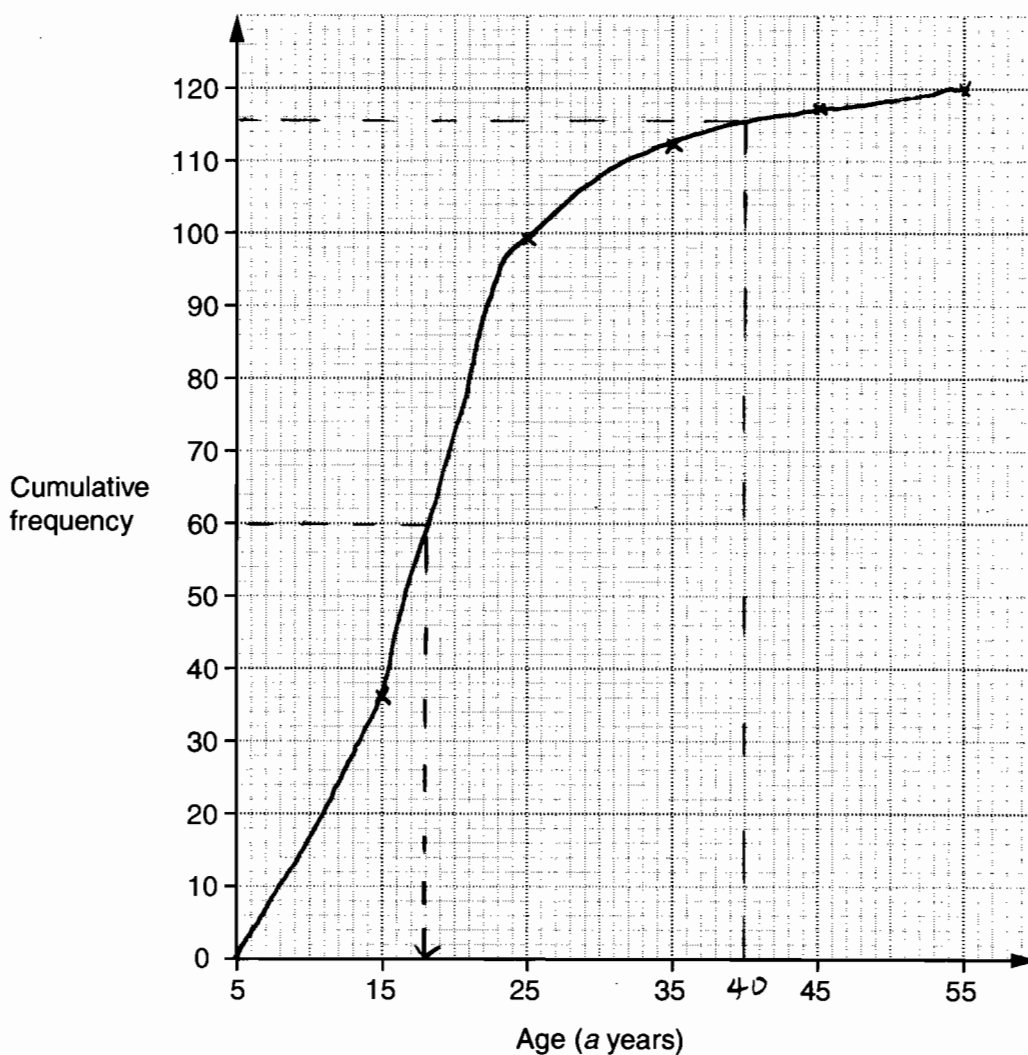
$$4\frac{1}{6}$$

[3]

15 This cumulative frequency table shows the distribution of the ages of the members of an athletics club.

Age (a years)	$a \leq 5$	$a \leq 15$	$a \leq 25$	$a \leq 35$	$a \leq 45$	$a \leq 55$
Cumulative frequency	0	36	99	112	117	120

(a) On the grid below, draw a cumulative frequency diagram for these ages.



[3]

(b) Use the cumulative frequency diagram to estimate the median age of the members.

Median at 60 out of 120 people

(b) 18 years [1]

(c) Geoff says "Not many of the members are over 40."

Explain why Geoff is correct.

The number up to 40 is 116, so only 4 above age 40

[1]

16 (a) Factorise and solve.

$$x^2 - 2x - 15 = 0$$

$$(x+3)(x-5) = 0$$

$$\begin{array}{ll} +1 & -15 \\ -1 & +15 \\ +3 & -5 \checkmark \\ -3 & +5 \end{array}$$

$$\text{Either } x+3=0 \quad \text{or} \quad x-5=0$$

$$\Rightarrow x = -3 \quad \Rightarrow x = +5$$

$$(a) \underline{x = -3, x = 5} \quad [3]$$

(b) Solve.

$$3x^2 - 12 = 0$$

$$\div 3$$

$$x^2 - 4 = 0$$

$$x^2 - 2^2 = 0$$

$$\text{Difference of two squares} \quad (x+2)(x-2) = 0$$

$$\text{Either } x+2=0 \quad \text{or} \quad x-2=0$$

$$\Rightarrow x = -2 \quad \Rightarrow x = 2$$

$$(b) \underline{x = 2, x = -2} \quad [3]$$

(c) Show that the equation $(2x-1)^2 = 9 - 6x^2$ can be simplified to

$$5x^2 - 2x - 4 = 0.$$

$$(2x-1)(2x-1) = 9 - 6x^2$$

$$4x^2 - 2x - 2x + 1 = 9 - 6x^2$$

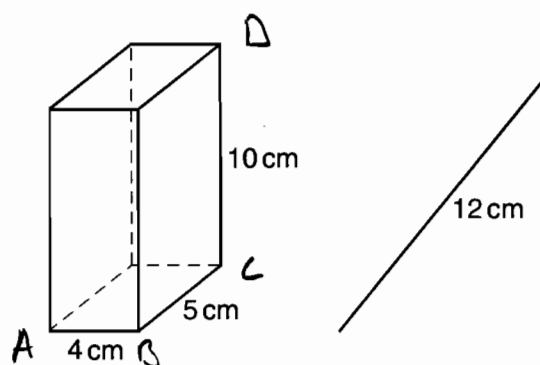
$$4x^2 + 6x^2 - 4x + 1 - 9 = 0$$

$$10x^2 - 4x - 8 = 0$$

$$\div 2 \quad 5x^2 - 2x - 4 = 0$$

[3]

- 17 An empty box is a cuboid with internal measurements 4 cm by 5 cm by 10 cm.



Is it possible to fit a thin, straight rod that is 12 cm long entirely inside the box?
Use calculations to show how you decide.

AD is longest possible length

Pythagoras

$$AC^2 = 4^2 + 5^2 = 16 + 25 = 41$$

Pythagoras

$$AD^2 = AC^2 + CD^2$$

$$AD^2 = 41 + 10^2 = 41 + 100 = 141$$

$$AD = \sqrt{141}$$

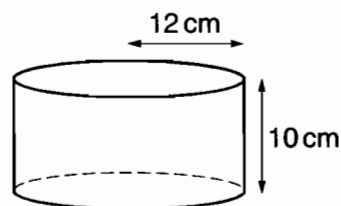
[4]

which is less than 12
because $12^2 = 144$

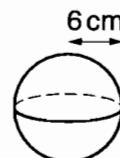
\therefore rod will not fit entirely
inside box

18 In this question, do not substitute a numerical value for π .

A solid metal cylinder has radius 12 cm and height 10 cm.



The cylinder is melted down and all of the metal made into solid spheres of radius 6 cm.



Work out how many of these spheres are made.
Show your working clearly.

$$\begin{aligned} \text{Vol of cylinder} &= \pi r^2 h = \pi \times 12^2 \times 10 \\ &= \pi \times 144 \times 10 \\ &= 1440\pi \end{aligned}$$

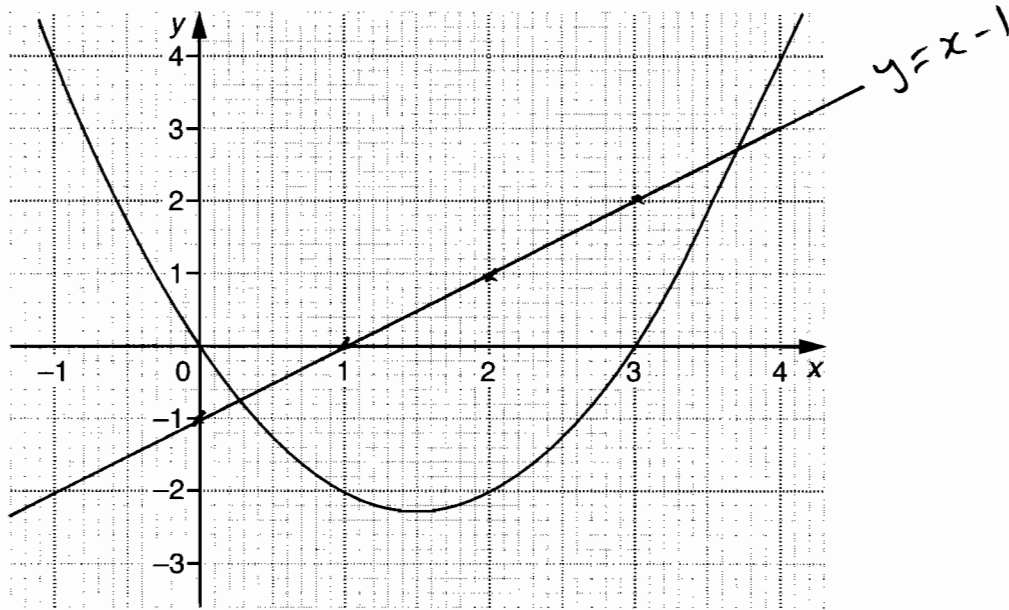
$$\begin{aligned} \text{Vol of sphere} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times 6^3 \\ &= \frac{4 \times \pi \times 6 \times 6 \times 6}{3} \\ &= 288\pi \end{aligned}$$

$$\text{Number of spheres} = \frac{1440\pi}{288\pi} = \frac{1440}{288}$$

$$\text{Cancel by 4} = \frac{360}{72} = \frac{90}{18} = 5$$

[6]

19 The diagram shows the graph of $y = x^2 - 3x$.



(a) By drawing a suitable straight line on this diagram, solve the equation

$$x^2 - 3x = x - 1.$$

Draw $y = x - 1$

From graphs intersecting

$$x = 0.3, x = 3.7$$

(a) $x = 0.3, x = 3.7$ [3]

(b) Find the equation of the line that would need to be drawn on the grid to solve the equation $x^2 - 5x + 2 = 0$.

$$x^2 - 3x = 2x - 2$$

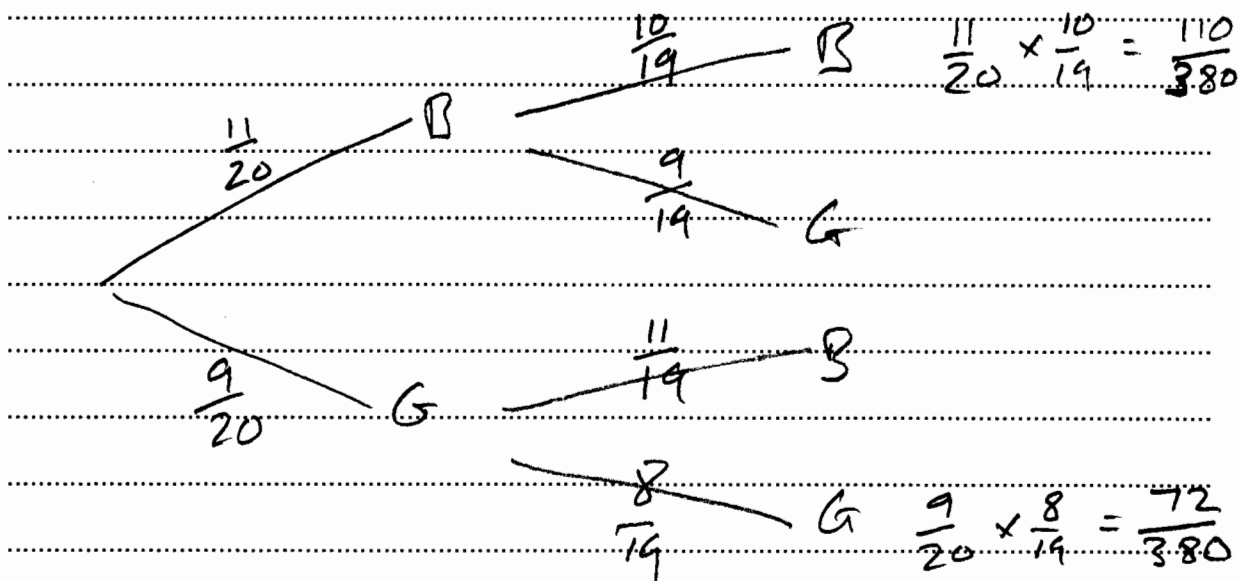
Draw $y = 2x - 2$

(b) $y =$ $2x - 2$ [2]

TURN OVER FOR QUESTION 20

- 20 There are 11 boys and 9 girls in a school maths club.
Two members of the club are chosen at random to represent the school in a competition.

Work out the probability that the two members chosen are of the same sex.



$$P(\text{BB or GG}) = \frac{110}{380} + \frac{72}{380} = \frac{182}{380}$$

[4]

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