OCR
RECOGNISING ACHIEVEMENT

H

GENERAL CERTIFICATE OF SECONDARY EDUCATION

MATHEMATICS SYLLABUS A

J512/04
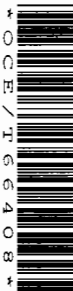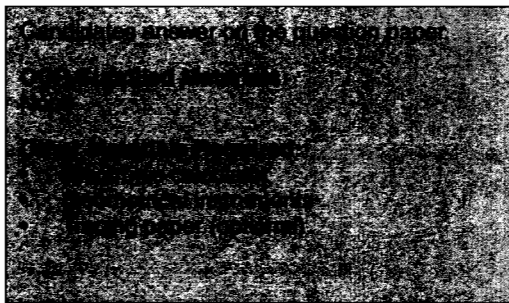
Paper 4
(Higher Tier)

Solutions

Monday 1 June 2009
Morning

Duration: 2 hours


INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Show your working. Marks may be given for a correct method even if the answer is incorrect.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are expected to use an electronic calculator for this paper.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- The total number of marks for this paper is **100**.
- This document consists of **20** pages. Any blank pages are indicated.

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Turn over

Formulae Sheet: Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$
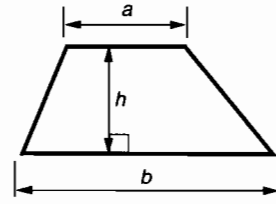
Volume of prism = (area of cross-section) x length
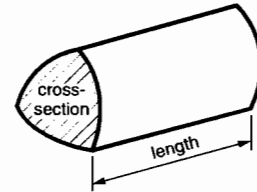
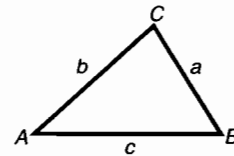**In any triangle *ABC***

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc \cos A$

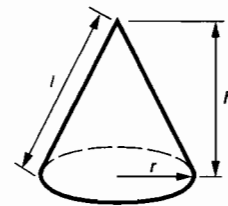**Area of triangle** $= \frac{1}{2} ab \sin C$

Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$
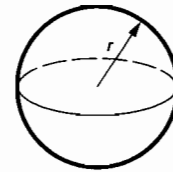
Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

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1 Joan has a recipe for making chocolate buns.
The recipe uses 150 g of flour and 25 g of cocoa powder to make 12 buns.

(a) How much flour should Joan use to make 18 buns?

12 buns → 150g flour

1 bun → 150 ÷ 12

18 buns → 150 ÷ 12 × 18 = 225

(a) _____ 225 _____ g [2]

(b) What is 25 g out of 175 g expressed as a percentage? $\frac{25}{175} \times 100 = 14.28\%$

(b) _____ 14.28 _____ % [2]

2 Calculate.

(a) $\left(\frac{22.4}{3.6 + 2.8}\right)$ = 3.5

(a) _____ 3.5 _____ [1]

(b) $\sqrt{36 + (4.5)^2}$ $= \sqrt{(36 + 4.5^2)}$ $= 7.5$

(b) _____ 7.5 _____ [2]

(c) The reciprocal of 0.16. $= \frac{1}{0.16} = 6.25$

(c) _____ 6.25 _____ [1]

Turn over

3   Rachael decides to bath her dog, Oscar. She runs water into the bath, puts Oscar in the bath, baths Oscar, takes him out and then empties the bath.
The graph shows the depth of water in the bath.

Water depth (cm) vs Time (minutes)
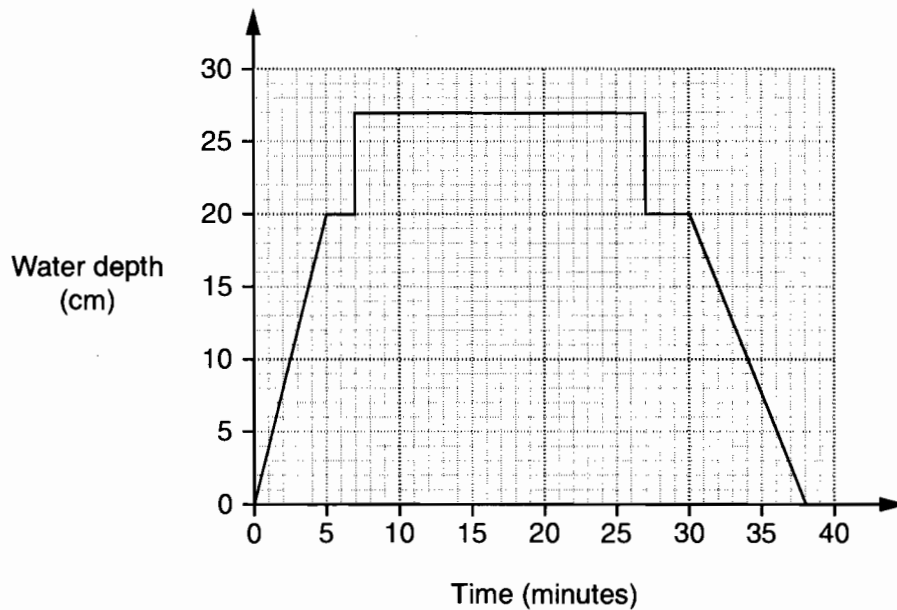
(a) Which is quicker, running water into the bath or emptying it?
Explain how you can tell.

    Running water into bath is quicker
    Tell by steeper slope of line water depth/time [1]

(b) By how much does the depth of the water increase in one minute as water runs into the bath?

    20 cm in 5 min
    ⇒ 4 cm in 1 min

(b) _____4_____ cm [1]

(c) For how long was Oscar in the bath?

    7 mins to 27 mins ⇒ in bath for 20 mins

(c) _____20_____ minutes [1]

(d) The volume of the bath water was 119 600 cm³.
    1000 cm³ = 1 litre

Change 119 600 cm³ into litres.

    119600 ÷ 1000 = 119.6 litres

(d) _____119.6_____ litres [1]

4   Calculate the average speed for a journey of 420 miles that takes 3 hours 30 minutes.

Av Speed = $\frac{Distance}{Time}$ = $\frac{420}{3.5}$ = 120 mph

_____120_____ mph **[3]**

5   Enlarge the triangle by scale factor 2 from centre **A**.
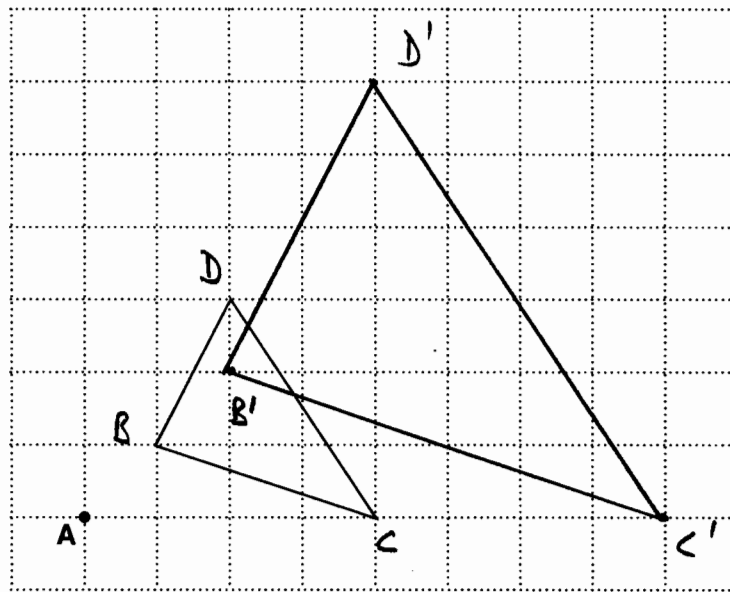
[3]

Double the distances of each
corner of Δ from A

So   C is 4 right of A ⇒ C' is 8 right of A

B is 1 right, 1 up from A ⇒ B' is 2 right, 2 up from A

D is 2 right, 3 up from A ⇒ D' is 4 right, 6 up from A

Turn over

6 Sally conducted an experiment on reaction times.
   One day ten students were woken at 4 am and their reaction times were measured.
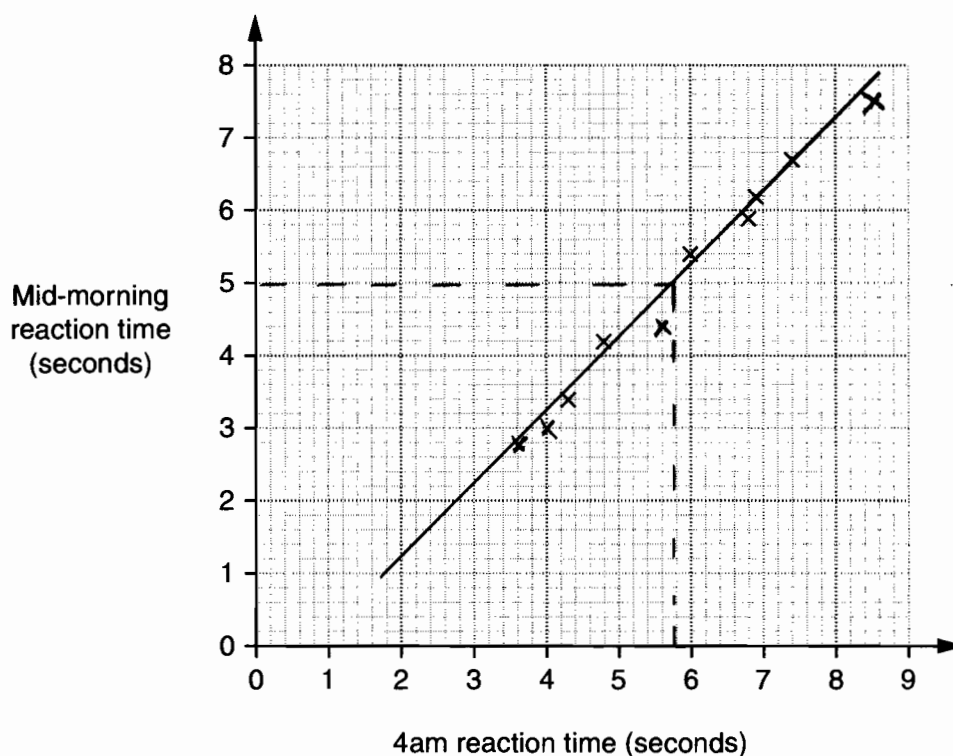   Another day their reaction times were measured mid-morning after a full night's sleep.

The reaction times, in seconds, are shown in the table.

Student		G	H	I	J
4 am reaction time		5.6	4.0	8.5	3.6
Mid-morning reaction time		4.4	3.0	7.5	2.8

The scatter graph shows the reaction times for students A to F.

4am reaction time (seconds)

(a) Complete the scatter graph.   [2]

(b) Describe the correlation shown.

        Strong   Positive   Correlation   [1]

(c) Draw a line of best fit on your graph.   [1]

(d) Another student, John, has a mid-morning reaction time of 5.0 seconds.

Use your line of best fit to estimate John's reaction time if he was woken at 4 am.

(d) _____ 5.8 _____ s [1]

(e) Explain why it may not be appropriate to use your line of best fit to estimate the mid-morning reaction time of a student whose 4 am reaction time is 2.0 seconds.
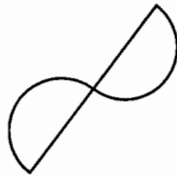
Outside the range of times for which
data is available _____ [1]

7 **(a)** Calculate the area of a circle of radius 6 cm. $Area = \pi r^2$

$= \pi \times 6 \times 6 = 113.1$

(a) _____ 113.1 _____ cm² [2]

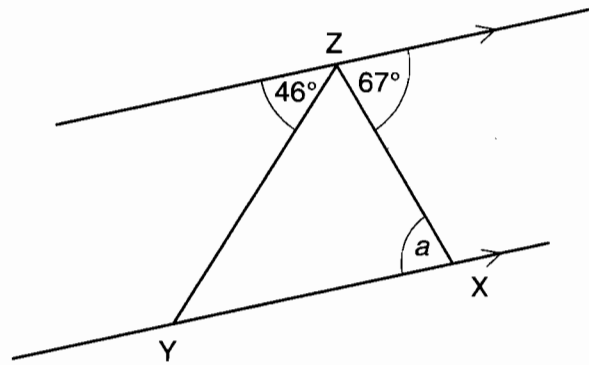**(b)** This logo is made using two semicircles, each with a radius of 6 cm.

Work out the **perimeter** of the logo.
Give the units of your answer.

Perimeter given by the full circle + 2 diameters

$= 2\pi r + 4r = 2 \times \pi \times 6 + 4 \times 6$ cm

(b) _____ 61.7 cm _____ [4]

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(a) Find the size of angle *a*.
Give a reason for your answer.

....

....

a = _____ 67 _____ ° because _____ alternate angles are equal

_____ **[2]**

(b) Find the sizes of the other angles in triangle XYZ.
What type of triangle is XYZ?

Angle Y = 46° (alternate angles)
Angle Z = 180° − (46 + 67)
           = 180° − 113° = 67°

Angles are _____ 46 _____ ° and _____ 67 _____ °.

Triangle XYZ is _____ isosceles _____ **[3]**

9 Eight identical squares have a total area of 2312 mm².
   The length of the side of each square is x mm.

   Write an equation in x and work out the value of x.

$$8x^2 = 2312$$
$$x^2 = \frac{2312}{8} = 289$$
$$x = \sqrt{289} = 17 \text{ mm}$$
$$\underline{x = 17} \qquad \text{[3]}$$

10 The capacity of a tank is 8 gallons.
    The empty tank was filled with petrol.
    The petrol cost 123.9 pence per litre.
    1 litre is approximately equal to 0.22 gallons.

    Calculate how much it cost to fill the tank with petrol.
    Give your answer to an appropriate degree of accuracy.

$$\text{Number of litres} = \frac{8}{0.22}$$

$$\text{Cost} = \text{Number of litres} \times \text{price per litre}$$

$$= \frac{8}{0.22} \times £1.239 = £45.05$$

£ __45.05__ [4]

Turn over

11 Use trial and improvement to solve this equation.

$$x^3 - x = 10$$

Give your answer to one decimal place.
Show all your trials and their outcomes.

x	$x^3 - x$		
2	$2^3 - 2$	=	6
3	$3^3 - 3$	=	24
2.5	$2.5^3 - 2.5$	=	13.125
2.4	$2.4^3 - 2.4$	=	11.424
2.3	$2.3^3 - 2.3$	=	9.867
2.35	$2.35^3 - 2.3$	=	10.627875

Answer between $x = 2.3$ and $x = 2.35$

so $x = 2.3$ to 1 d.p.

_____ $x = 2.3$ _____ [4]

12 A fair spinner is numbered from 1 to 5.
The spinner is spun twice.

By listing the possible outcomes, or otherwise, work out the probability that the number obtained on the second spin is less than the number obtained on the first spin.

11	21 ✓	31 ✓	41 ✓	51 ✓
12	22	32 ✓	42 ✓	52 ✓
13	23	33	43 ✓	53 ✓
14	24	34	44	54 ✓
15	25	35	45	55

Prob (2nd spin less than 1st) = $\dfrac{10}{25}$ or $\dfrac{2}{5}$

_____ $\dfrac{2}{5}$ _____ [3]

13 A bouncy ball is dropped from a height of 4 m.
After each bounce it rises to 90% of its previous height.

(a) To what height does it rise after one bounce?

$4 m \times 0.9 = 3.6 m$

(a) _____ 3.6 _____ m **[2]**

(b) How many times does it bounce altogether before the height it rises to is less than 2.5 m?

2nd $4 \times 0.9^2 = 3.24$ 5th $4 \times 0.9^5 = 2.36196$

3rd $4 \times 0.9^3 = 2.916$

4th $4 \times 0.9^4 = 2.6244$

(b) _____ 5 bounces _____ **[2]**

14 In this question the letters f, g, and h represent lengths.

(a) Here are some expressions.

$$f^2gh \qquad f^2(g + h) \qquad 2(f + g) \qquad 2h(f - g)$$

Write down the expression that represents

(i) a length,

(a)(i) _____ $2(f + g)$ _____ **[1]**

(ii) a volume.

(ii) _____ $f^2(g + h)$ _____ **[1]**

(b) Explain why $f + g^2$ cannot represent a length, an area or a volume.
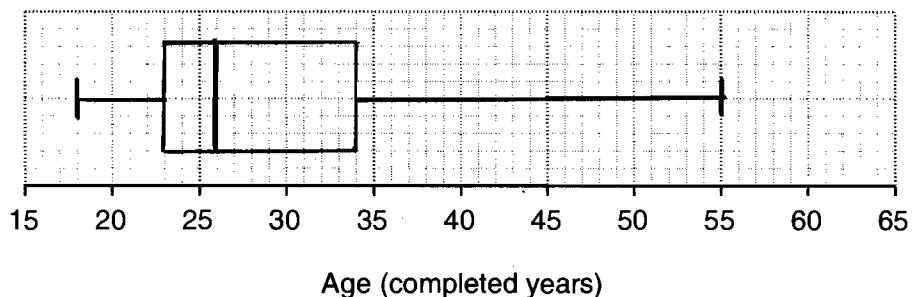
Cannot add a length to an area

_____ **[1]**

Turn over

15 Ajay carried out a survey to find the age, in completed years, at which people marry.

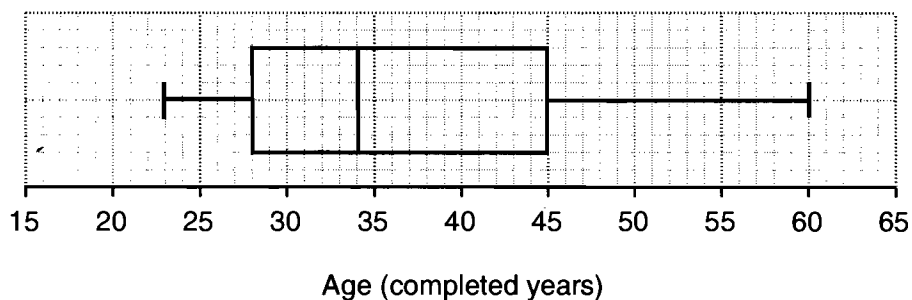The table summarises the results for the women in the survey.

	Age (completed years)
Youngest	18
Lower quartile	23
Median	26
Upper quartile	34
Oldest	55

(a) Draw a box plot to summarise the results for the women in the survey.

Age (completed years)

[2]

The box plot below summarises the results for the men in the survey.

Age (completed years)

(b) Give one similarity and one difference between the ages at which the men and the women in the survey marry.

Similarity They have Similar range. In fact both men and women have a range of 37 years

Difference Men have greater median

or Men have greater inter-quartile range [2]

(c) Ajay decides to interview two people from his survey.
He chooses one woman and one man at random.

What is the probability that both the woman and the man were aged 34 or more when they married?

$\text{Prob}(\text{Man} \geqslant 34) = 0.25$, $\text{Prob}(\text{Woman} \geqslant 34) = 0.5$

$\text{Prob}(\text{Both} \geqslant 34) = 0.25 \times 0.5 = 0.125$

or $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = 0.125$

(c) _____ 0.125 _____ **[2]**

16 (a) Solve, algebraically, these simultaneous equations.

$$3x - 5y = 111 \quad ①$$
$$4x + 3y = -26 \quad ②$$

① $\times 3$ $\quad 9x - 15y = 333 \quad ③$

② $\times 5$ $\quad 20x + 15y = -130 \quad ④$

Sub for x in ②

③+④ $\quad 29x = 203$

$x = \dfrac{203}{29} = 7$

$4(7) + 3y = -26$

$28 + 3y = -26$

$3y = -26 - 28$

$3y = -54$

$y = -\dfrac{54}{3} = -18$

(a) $x =$ _____ $y =$ _____ **[4]**

(b) Rearrange $t = \sqrt{\dfrac{2V}{r}}$ to make V the subject of the formula.

$x = 7 \quad y = -18$

$t^2 = \dfrac{2V}{r}$

$t^2 r = 2V$

$\dfrac{t^2 r}{2} = V$

(b) _____ $V = \dfrac{t^2 r}{2}$ _____ **[3]**

Turn over

17 ABCDE is an irregular pentagon with lengths and angles as shown in the diagram.
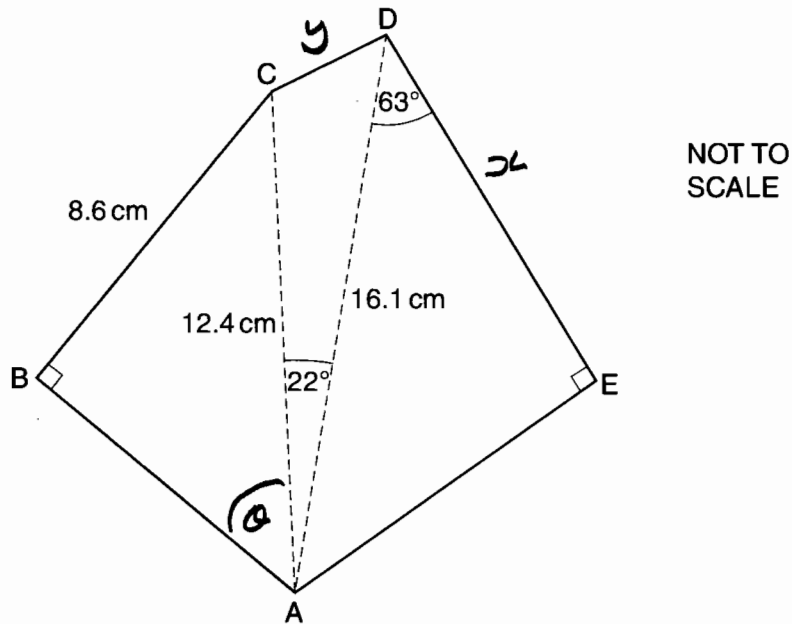
(a) Work out the size of angle CAB. $\sin = \frac{O}{H}$

$$\sin \theta = \frac{8.6}{12.4}$$

$$\theta = \sin^{-1}\left(\frac{8.6}{12.4}\right)$$

$$\theta = 43.9°$$

(a) _____ 43.9 _____ ° [3]

(b) Work out the length DE.

$$\cos = \frac{A}{H}$$

$$\cos 63° = \frac{x}{16.1}$$

$$16.1 \cos 63° = x$$

$$x = 7.31 \text{ cm}$$

(b) _____ 7.31 _____ cm [3]

(c) Work out the area of triangle ACD.

$$\text{Area} = \tfrac{1}{2} ab \sin C$$

$$= \tfrac{1}{2} \times 12.4 \times 16.1 \times \sin 22°$$

$$= 37.4 \text{ cm}^2$$

(c) ____37.4____ cm² **[2]**
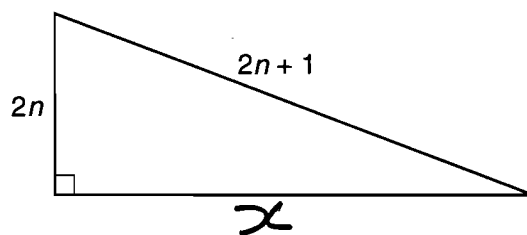
(d) Work out the length CD.

Cosine rule

$$y^2 = 12.4^2 + 16.1^2 - 2 \times 12.4 \times 16.1 \times \cos 22°$$

$$y^2 = 42.764$$

$$y = \sqrt{42.764} \quad = 6.54 \text{ cm}$$

(d) ____6.54____ cm **[3]**

Turn over

18 A right-angled triangle has hypotenuse of length $2n + 1$, where n is an integer.
Another side has length $2n$.

(a) Show that the length of the third side of the triangle is $\sqrt{4n + 1}$.

By Pythagoras

$$x^2 + (2n)^2 = (2n+1)^2$$

$$x^2 + (2n)^2 = 4n^2 + 4n + 1$$

$$x^2 = 4n^2 + 4n + 1 - 4n^2$$

$$x^2 = 4n + 1$$

$$x = \sqrt{4n+1}$$

[3]

(b) For some values of n, all the sides of the triangle are integer lengths.

For these values of n, is $\sqrt{4n + 1}$ odd or even?
Explain your answer.

Must be odd

Need $odd^2 + even^2$ to give odd^2

and $(2n+1)^2$ is odd^2

and $(2n)^2$ is $even^2$

[2]

OR   whether n is odd or even

$4n$ is even

So $4n+1$ is odd

So $\sqrt{4n+1}$ is odd if it is

an integer

19 Pond weed was introduced into a pond.
The weight of pond weed, p grams, in the pond t days later is given by

$$p = 1 + 2^t.$$

(a) Work out the weight of pond weed in the pond after 3 days.

$p = 1 + 2^3 = 1 + 8 = 9 \text{ grams}$

(a) _____9_____ g [1]

(b) (i) Write down an equation that could be used to find the number of days it takes for the weight of pond weed to reach 129 grams.

$129 = 1 + 2^t$

(b)(i) ____$129 = 1 + 2^t$____ [1]

(ii) Work out the number of days it takes for the weight of pond weed to reach 129 grams.
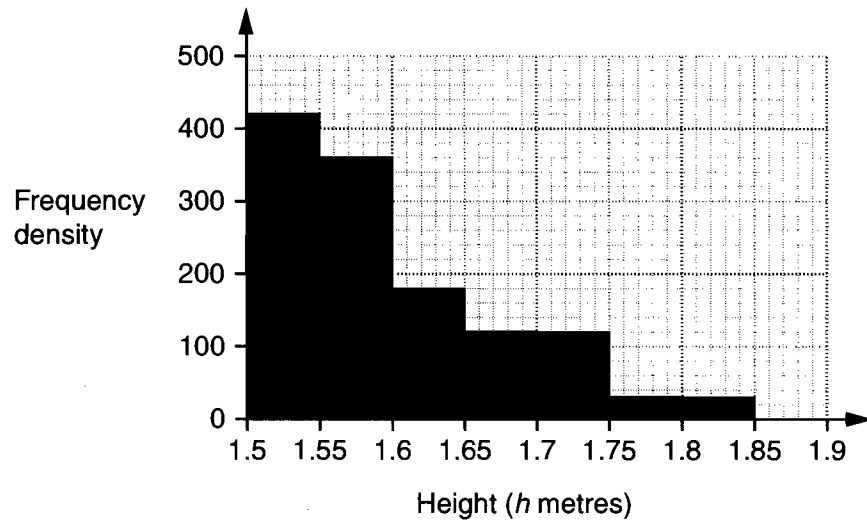
$129 = 1 + 2^t$
$129 - 1 = 2^t$
$128 = 2^t$

$128 = 2^7$

(ii) ____$t = 7 \text{ days}$____ [2]

Turn over

20 The histogram shows the distribution of the heights, h metres, of a group of nurses.

Frequency density (y-axis) vs Height (h metres) (x-axis)

(a) Show that 12 of these nurses have height $1.65 \leqslant h < 1.75$.

$\underline{Freq = Freq \; Density \times Width = 120 \times 0.1 = 12}$

_____ [1]

(b) Complete the frequency table.

Height (h metres)	Frequency
$1.50 \leqslant h < 1.55$	21
$1.55 \leqslant h < 1.60$	18
$1.60 \leqslant h < 1.65$	9
$1.65 \leqslant h < 1.75$	12
$1.75 \leqslant h < 1.85$	3

420×0.05

360×0.05

180×0.05

30×0.1

....

....

.... [3]

21 Solve algebraically.

$$\frac{5}{x+3} + \frac{4}{2x-1} = 1$$

Give your answers correct to 2 decimal places.

$$5(2x-1) + 4(x+3) = 1(x+3)(2x-1)$$

$$10x - 5 + 4x + 12 = 2x^2 + 6x - x - 3$$

$$0 = 2x^2 + 5x - 3 - 10x - 4x + 5 - 12$$
$$0 = 2x^2 - 9x - 10$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{9 \pm \sqrt{81 + 80}}{4} = \frac{9 \pm \sqrt{161}}{4}$$

$$x = 5.42 \quad \text{or} \quad x = -0.92$$

$$\underline{x = 5.42 \, , \, x = -0.92} \quad \text{[7]}$$

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