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RECOGNISING ACHIEVEMENT

H

Wednesday 13 June 2012 – Morning

GCSE MATHEMATICS B

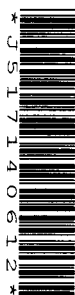J567/04   Paper 4 (Higher Tier)

$\int o l u t i o n s$

Candidates answer on the Question Paper.
OCR supplied materials: None
Other materials required: • Geometrical instruments • Tracing paper (optional) • Scientific or graphical calculator

Duration: 1 hour 45 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Your answers should be supported with appropriate working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- Your Quality of Written Communication is assessed in questions marked with an asterisk (*).
- The total number of marks for this paper is **100**.
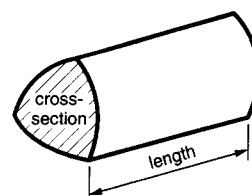- This document consists of **20** pages. Any blank pages are indicated.

⬛ You are permitted
to use a calculator
for this paper
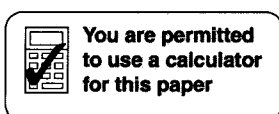
This paper has been pre modified for carrier language

Turn over

Formulae Sheet: Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$
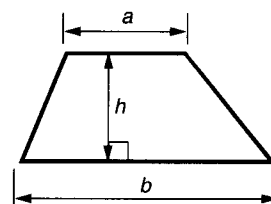
Volume of prism = (area of cross-section) × length

In any triangle ABC

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$
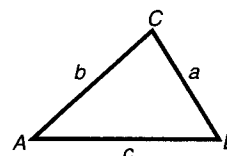
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
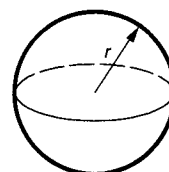
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
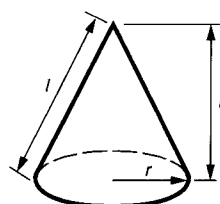
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

1 **(a)** Work out the value of $u + 5t$ when $u = {}^-16.4$ and $t = 3.5$.

$$-16.4 + 5 \times 3.5$$

(a) _____ 1.1 _____ **[2]**

(b) Rearrange $v = u + 5t$ to make t the subject.

$$V - u = 5t$$

$$\frac{V-u}{5} = t$$

$$t = \frac{V-u}{5}$$

(b) _____ **[2]**

2 **(a)** Write 600 as a product of its prime factors.

```
2 | 600
2 | 300
2 | 150
3 | 75
5 | 25
5 | 5
  | 1
```

$$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$$

(a) $2 \times 2 \times 2 \times 3 \times 5 \times 5$ **[3]**

**(b)** At Rumblestone Station northbound trains stop every 20 minutes and southbound trains stop every 16 minutes.
Two trains stopped together at the station at 15 00.

Work out the next time when two trains will stop together at this station.

```
1500          1500
1520          1516
1540          1532
1600          1548
1620 ✓        1604
1640          1620 ✓
```

(b) _____ 1620 hrs _____ **[3]**

Turn over

3   Parvinder has a bicycle.
     Each wheel has a diameter of 65.5 cm.
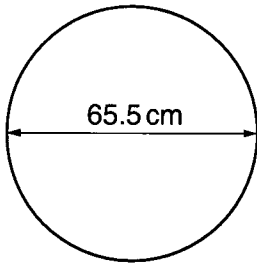
65.5 cm

$\text{diameter} = 65.5 \text{ cm}$

$= 0.655 \text{ m}$

$\text{Circumference} = \pi d$

$= \pi \times 0.655 \text{ m}$

On one journey each wheel rotated 3509 times.

Calculate the distance Parvinder cycled.
Give your answer in kilometres.

$\text{Distance} = 3509 \times \pi \times 0.655 \text{ m}$

$= 7220.62 \text{ m}$

$= 7.22 \text{ Km} \quad \text{to } 3 \text{ s.f.}$

_____7.22_____ km [4]

4 **(a)** The label on a sports drink states that each serving supplies 185 calories.
This is 9% of an adult's guideline daily amount (GDA).

Using this information, work out an adult's GDA.

$$9\% = 185$$

$$1\% = \frac{185}{9}$$

$$100\% = \frac{185}{9} \times 100 = 2055.56$$

(a) _____2056_____ calories **[2]**

(b) Betterbuy supermarket sells 3 brands of low-sodium salt.
The table shows, for each brand, how much sodium is in a packet.

Brand	Weight of packet (g)	Amount of sodium (g)
Low-salt	350	72
Salt-what-salt?	500	96
Salt Lite	1200	236

$$\%$$

$$\frac{72}{350} \times 100 = 20.6\%$$

$$\frac{96}{500} \times 100 = 19.2\%$$

$$\frac{236}{1200} \times 100 = 19.7\%$$

Which brand has the lowest proportion of sodium?
Show your working clearly.

Salt-what-salt   19.2 %   is   lowest

(b) _____Salt - what - salt ?_____ **[3]**

5 Ahmed grows 12 tomato plants.
He feeds each plant once with a different amount of fertiliser.
He records these amounts and the height to which each plant grows.

Amount of fertiliser (g)	36	58	14	26	10	43	52	40	18	50	55	30
Height of plant (cm)	37	66	24	35	32	46	69	54	33	47	58	42
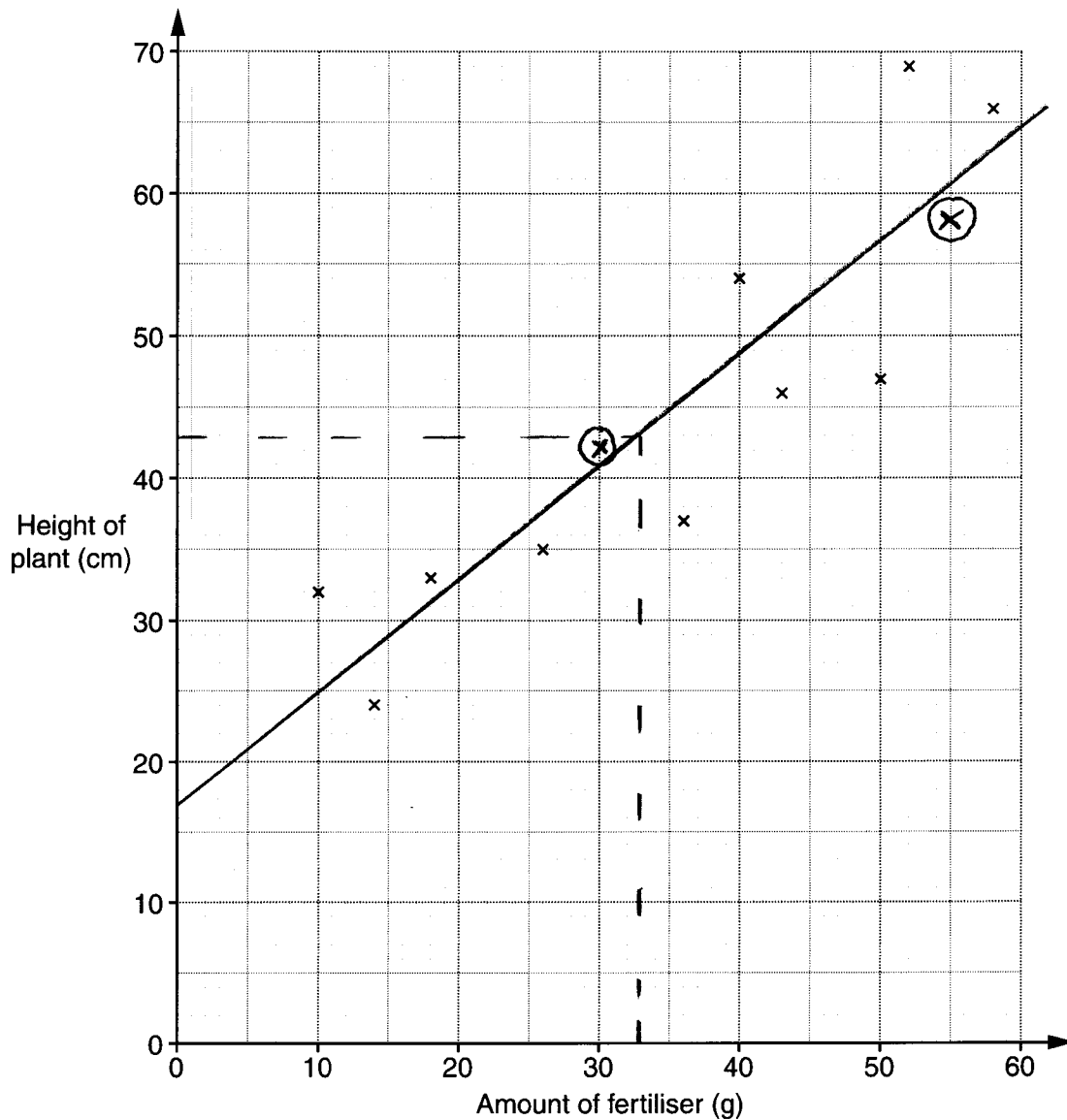
(a) The first 10 points have been plotted.

Complete the scatter diagram by plotting the last two points.   [1]

(b) State the correlation shown.

Positive correlation

_____ [1]

(c) (i) Draw a line of best fit on the diagram. [1]

(ii) Estimate the height of a tomato plant which was given 33 g of fertiliser.

(c)(ii) $\underline{\qquad 43 \qquad}$ cm [1]

6 (a) Lee completes a marathon in 2 hours 30 minutes. $= 2.5 \ hrs$
The marathon is 42.195 km long.

Calculate Lee's average speed. $speed = \dfrac{distance}{time}$

$$= \frac{42.195}{2.5}$$

$$= 16.878 \ Km/h$$

(a) $\underline{\qquad 16.878 \qquad}$ km/h [3]

(b) Calculate.

$$\sqrt{\frac{16.4 - 5.87}{5.42 + 1.09}}$$

Write your answer correct to 2 decimal places.

$= \sqrt{\left(\dfrac{(16.4 - 5.87)}{(5.42 + 1.09)}\right)}$ $= 1.27181$

(b) $\underline{\qquad 1.27 \qquad}$ [2]

Turn over

7   Jenny is doing a survey of the athletes at her club.

(a) Here is one of her questions.

> How many competitions have you entered during the past 12 months?
> *(Please tick one of the boxes)*
>
> 1 - 4 ☐   5 - 8 ☐   9 - 12 ☐   13 - 16 ☐

Make one criticism of this question.

No option for 0

_____

_____ [1]

(b) Jenny wants to find out how many hours the athletes train at the weekend.

Write a suitable question for Jenny to use to find this out.
Remember to include response boxes.   [2]

How many hours do you train over a weekend?

☐ 0   ☐ 1   ☐ 2   ☐ 3   ☐ 4   ☐ 5 or more

(c) Jenny will give the survey to a sample of athletes in her club.
Some information about the 175 members in her club is shown in this table.

	Female	Male
Junior	16	31
Senior	53	75

(i) **Explain** how to take a representative stratified sample.

The number sampled from each class should be proportional to the size of the class

_____ **[2]**

(ii) Jenny will use a sample size of 30.   Sampling fraction $\dfrac{30}{175}$

How many junior males should be included in her stratified sample?

31 Junior Males

Sample $31 \times \dfrac{30}{175} = 5.49$ so 5

(c)(ii) _____ 5 _____ **[2]**

(d) Jenny is a javelin thrower.
Here is a summary of the lengths of 40 of Jenny's throws this year.

Length of throw (s metres)	Frequency	Midpoint	Freq × Midpoint
$40 \leqslant s < 46$	4	43	172
$46 \leqslant s < 52$	12	49	588
$52 \leqslant s < 58$	19	55	1045
$58 \leqslant s < 64$	5	61	305
	40		2110

Calculate an estimate of the mean length of her javelin throws.

Estimate for mean $= \dfrac{2110}{40} = 52.75$
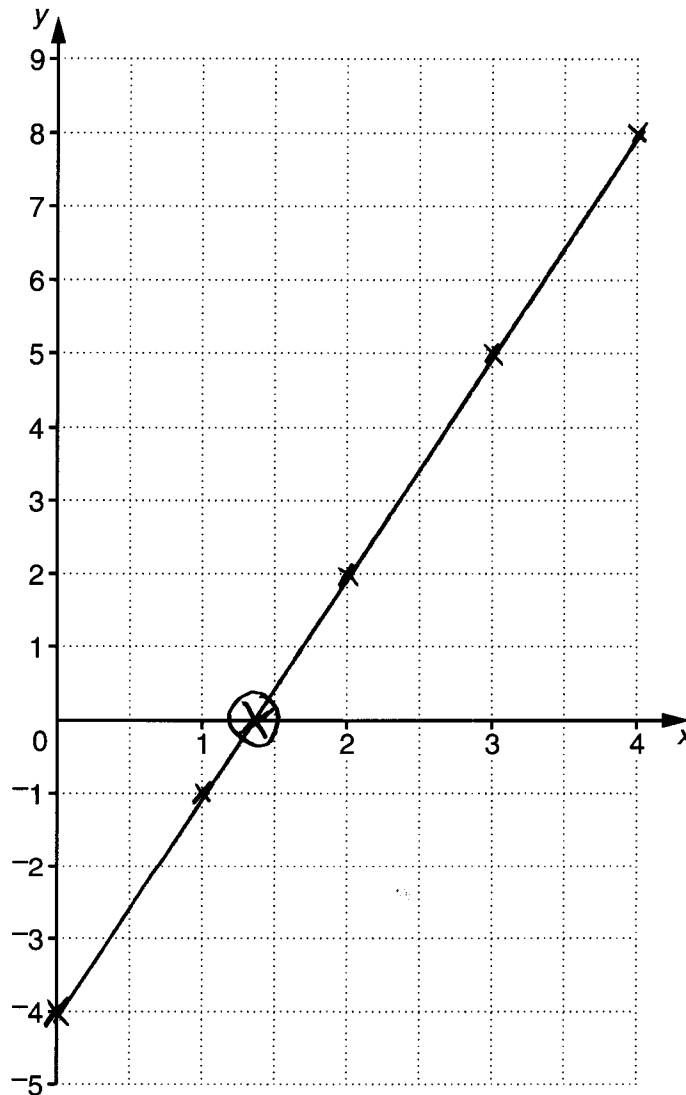
(d) _____ 52.75 _____ m **[4]**

$$3(0) - 4 = -4$$
$$3(2) - 4 = 2$$

8 **(a)** Complete this table for $y = 3x - 4$.

x	0	1	2	3	4
y	-4	$^-1$	2	5	8

[1]

(b) Plot these points on the grid and draw the graph of $y = 3x - 4$.

[2]

(c) On your graph put a cross (✗) at the point where $3x - 4 = 0$. [1]

(d) Write down the gradient of the line $y = 3x - 4$.

(d) _____ 3 _____ [1]

(e) Write down the equation of a line parallel to $y = 3x - 4$.

(e) $y = 3x + 1$ _____ [1]

or $y = 3x +$ any number

9 The equation $x^3 - 5x - 6 = 0$ has a solution between $x = 2$ and $x = 3$.

Find this solution correct to 1 decimal place.
Show clearly your trials and the values of their outcomes.

x	$x^3 - 5x - 6$	too big	too small
2	⁻8		✓
3	6	✓	
2.5	−2.875		✓
2.7	0.183	✓	
2.6	−1.424		✓
2.65	−0.640375		✓
So round up to 2.7			

You may not need to use all the rows.

$x = $ _____ 2.7 _____ [3]

10 The population of a species of bird is estimated using the formula

$N = 6400 \times (0.85)^t$

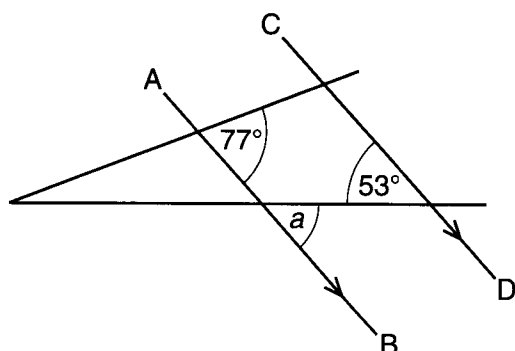where N is the number of birds
and t is the number of years after 2012.

Estimate the number of birds in 2015.

In 2015 $t = 3$

$N = 6400 \times 0.85^3 = 3930.4$

_____ 3930 _____ [2]

Turn over

11 (a) In the diagram below, AB is parallel to CD.

Not to scale

Complete this sentence.

Angle a = ___**53**___ ° because ___*alternate angles are equal*___

_____ [2]

(b) In the diagram, A and B are points on the circumference of a circle, centre O.
PA and PB are tangents to the circle.
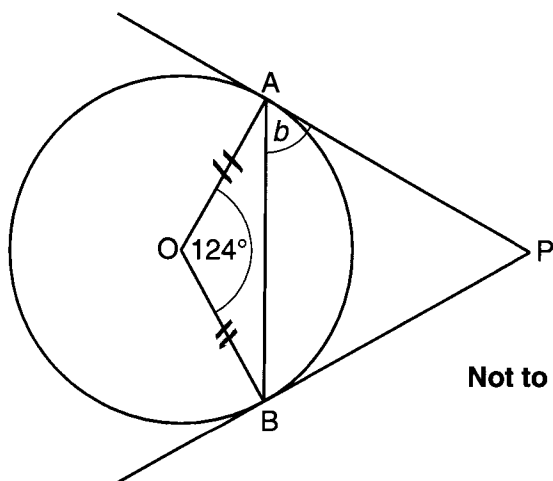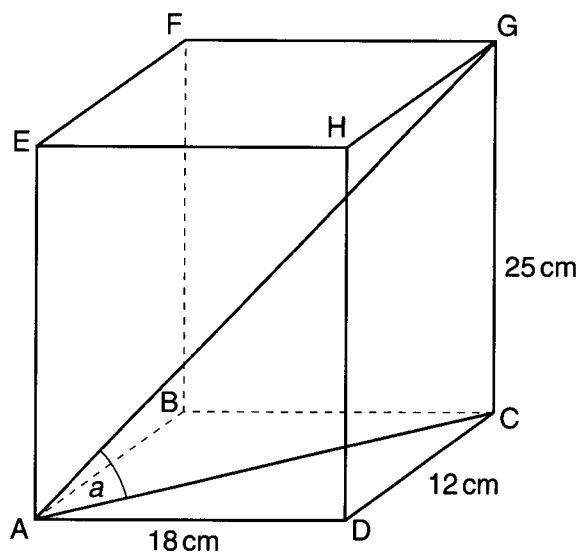

Not to scale

Calculate angle b.

$\angle OAB = \dfrac{180 - 124}{2} = 28°$

$b = 90 - 28 = 62°$

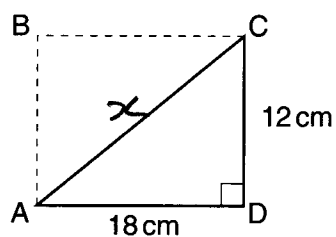(angle between tangent and radius = $90°$)

(b) ___**62**___ ° [2]

12 ABCDEFGH is a cuboid.

(a) Here is a diagram of the base.

12 cm   **Not to scale**

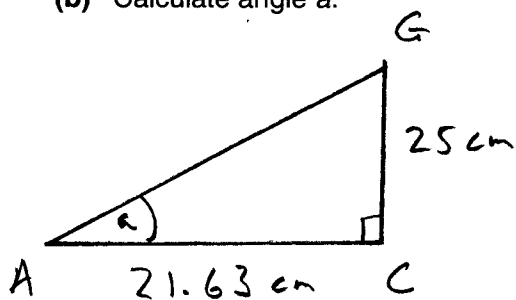Calculate the length of the diagonal AC.

$$x^2 = 12^2 + 18^2$$
$$x^2 = 468$$
$$x = \sqrt{468}$$
$$x = 21.63 \text{ cm}$$

(a) _____ 21.63 _____ cm **[3]**

(b) Calculate angle *a*.

$$\tan a = \frac{25}{21.63}$$

$$a = \tan^{-1}\left(\frac{25}{21.63}\right)$$

(b) _____ $a = 49.1$ _____ ° **[3]**

Turn over

13 (a) Solve.

$$\frac{6x-1}{4} = x + 5$$

$6x - 1 = 4(x+5)$

$6x - 1 = 4x + 20$

$6x - 4x = +20 + 1$

$2x = 21$

$x = \frac{21}{2}$

$x = 10\frac{1}{2}$

(a) $x =$ _____ $10\frac{1}{2}$ _____ **[3]**

(b) Expand and simplify.

$(x-6)(x+6) = x^2 - 6x + 6x - 36$

(b) _____ $x^2 - 36$ _____ **[2]**

(c) Factorise and solve.

$x^2 - 4x - 32 = 0$

$(x+4)(x-8) = 0$

$+1 \quad -32$
$-1 \quad +32$
$+2 \quad -16$
$-2 \quad +16$
$+4 \quad -8 \checkmark$

Either $x + 4 = 0$

$\Rightarrow x = -4$

or $x - 8 = 0$

$\Rightarrow x = 8$

(c) _____ $x = -4, \quad x = 8$ _____ **[3]**

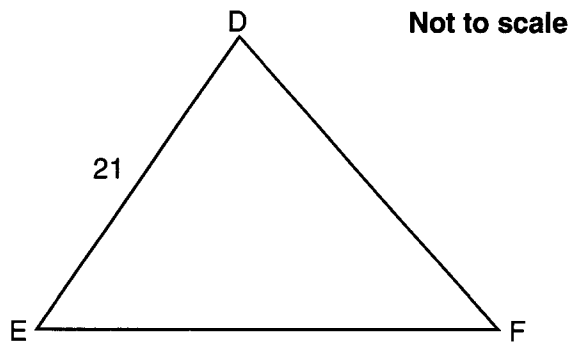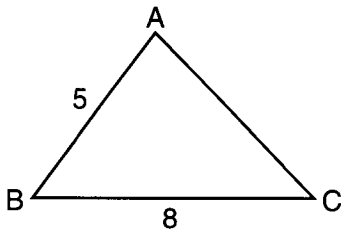**(d)** Rearrange this formula to make r the subject.

$$S = 4\pi r^2$$

$$\frac{S}{4\pi} = r^2$$

$$\sqrt{\frac{S}{4\pi}} = r$$

(d) $r = \sqrt{\frac{S}{4\pi}}$ _____ [2]

14 Triangles ABC and DEF are similar.
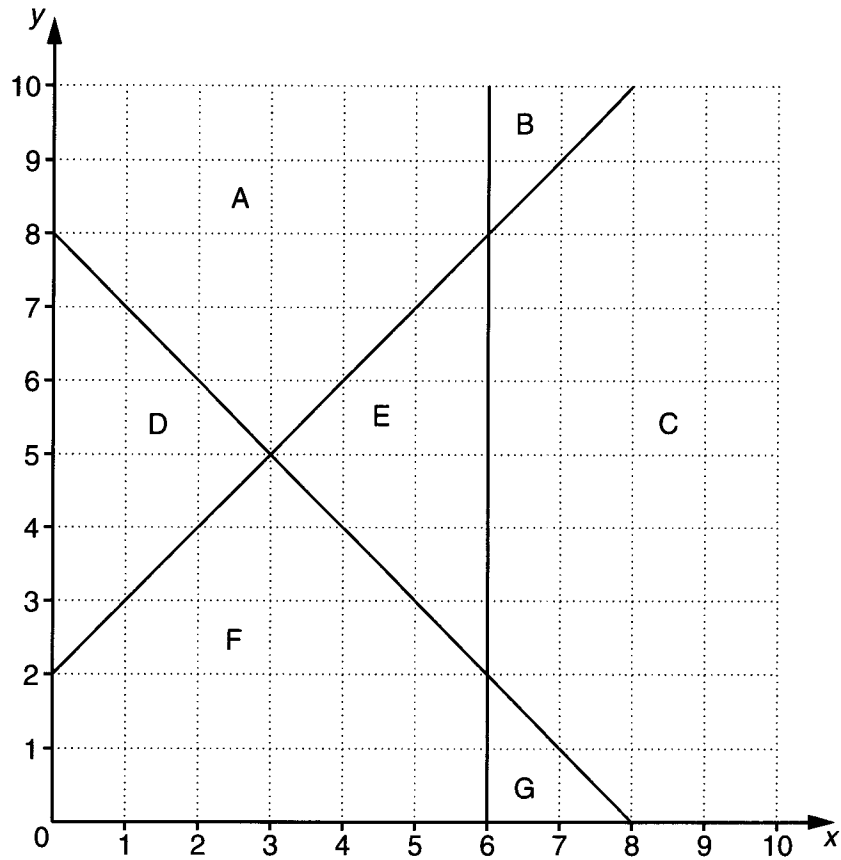All lengths are in centimetres.

Not to scale

Calculate EF. Scale   5 : 21

$$EF = 8 \times \frac{21}{5} = 33.6$$

_____ 33.6 _____ cm **[3]**

Turn over

15 A, B, C, D, E, F and G are regions on a coordinate grid.

(a) Write down the letters of **all** the regions which satisfy the inequality $x \geqslant 6$.

(a) _____B, C, G_____ [1]

(b) The regions D, F and G satisfy a different inequality.

Write down this inequality.

(b) _____$x + y \leqslant 8$_____ [2]

16 **(a)** In the 2004 Olympic Games, Great Britain (GB) won 30 medals.
In the 2008 Olympic Games, GB won 47 medals.

Calculate the percentage increase in the number of medals
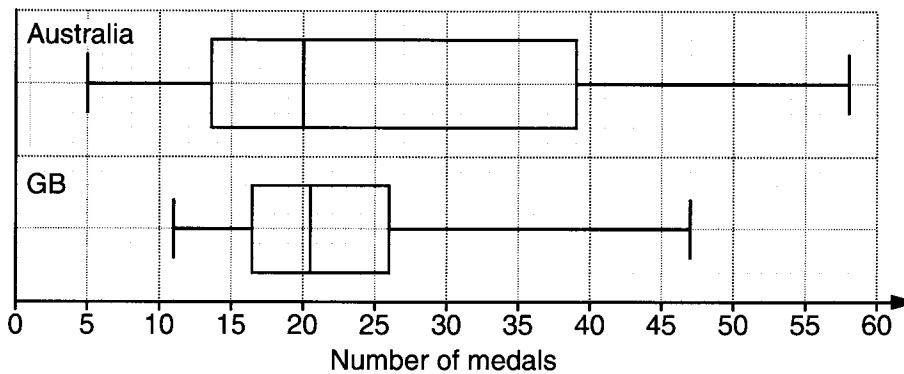won by GB from 2004 to 2008.

$$Increase = 47 - 30 = 17$$

$$Express\ 17\ as\ a\ percentage\ of\ 30$$

$$\frac{17}{30} \times 100 = 56.67\%$$

(a) ___56.7___ % **[3]**

(b) The box plots summarise the number of medals won by Australia and GB
at each Olympic Games since 1948.

**(i)** Work out the interquartile range for GB. $26 - 16.5 = 9.5$

(b)(i) ___9.5___ **[2]**

(ii) Make one comment comparing the numbers of medals won by Australia and GB.

Australia and GB had similar median

or Australia had much greater IQR than GB **[1]**

17 The diagram shows a container in the shape of a cone.
The radius of the top is 8 cm and the vertical height is 24 cm.
Both of these measurements are correct to the nearest centimetre.

8 cm

24 cm
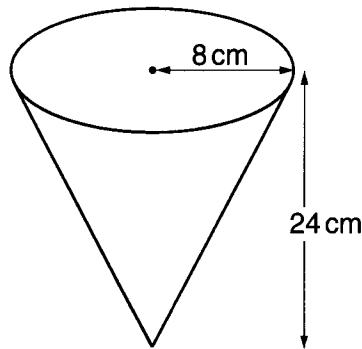
Upper bounds

radius $r = 8.5$ cm

height $h = 24.5$ cm

Calculate the upper bound for the volume of this cone.

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Upper bound} = \frac{1}{3} \times \pi \times 8.5^2 \times 24.5$$

$$= 1853.670565$$

_____ 1853.67 _____ cm³ **[4]**

18 The curve $y = x^2$ is translated by $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

Write down the equation of the transformed curve.

$$y = (x+3)^2 + 2$$

_____ **[2]**

19 Solve these simultaneous equations.
  Give your answers correct to 2 decimal places.

$$y^2 = 3x + 2 \qquad ①$$
$$y = 2x - 1 \qquad ②$$

Sub for y in ①

$$(2x-1)^2 = 3x+2$$

$$(2x-1)(2x-1) = 3x+2$$

$$4x^2 - 2x - 2x + 1 = 3x + 2$$

$$4x^2 - 7x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{49 - 4(4)(-1)}}{8}$$

$$x = \frac{7 \pm \sqrt{65}}{8}$$

$$x = \frac{7 + \sqrt{65}}{8} \qquad \text{or} \qquad x = \frac{7 - \sqrt{65}}{8}$$

$$x = 1.88 \qquad\qquad\qquad x = -0.13$$

Sub for x in ②

$$y = 2 \times 1.88 - 1 \qquad\qquad y = 2x - 0.13 - 1$$

$$y = 2.76 \qquad\qquad\qquad y = -1.26$$

$x =$ _____1.88_____ $y =$ _____2.76_____

$x =$ _____-0.13_____ $y =$ _____-1.26_____ **[6]**

TURN OVER FOR QUESTION 20

20* This is a diagram of a field where Lucy wants to keep horses.

A

265 m

168 m

B

304 m

C

Not to scale
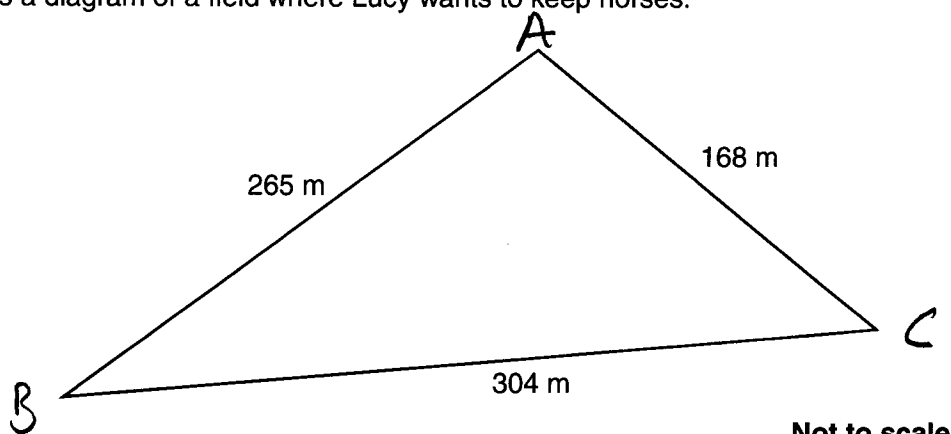
Each horse must have one acre of field.
One acre is equivalent to 4046.856 m².

What is the largest number of horses Lucy can keep in this field?

Need area of $\triangle$ using $\frac{1}{2}ab \sin C$

First need an angle using cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc \cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(265^2 + 168^2 - 304^2)}{(2 \times 265 \times 168)}$$

$$\cos A = 0.067756$$

$$A = \cos^{-1}(0.067756)$$

$$A = 86.11°$$

Area $= \frac{1}{2}bc \sin A = \frac{1}{2} \times 168 \times 265 \times \sin 86.11°$

$$= 22208.7 \, m^2$$

Number of horses $= \frac{22208.7}{4046.856} = 5.49$ horses

So limited to 5 horses

5

[6]