A* 83 , A 66 , B 48 , C 30

OCR
RECOGNISING ACHIEVEMENT

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Monday 4 March 2013 – Morning

GCSE MATHEMATICS B

J567/04   Paper 4 (Higher Tier)

Solutions

Duration: 1 hour 45 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

* Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Answer **all** the questions.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Your answers should be supported with appropriate working. Marks may be given for a correct method even if the answer is incorrect.
* Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

* The number of marks is given in brackets **[ ]** at the end of each question or part question.
* Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
* Your Quality of Written Communication is assessed in questions marked with an asterisk (*).
* The total number of marks for this paper is **100**.
* This document consists of **24** pages. Any blank pages are indicated.
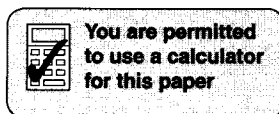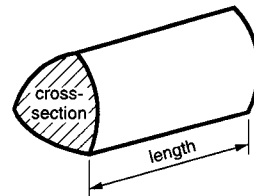
This paper has been pre modified for carrier language

OCR is an exempt Charity

Turn over

Formulae Sheet: Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$
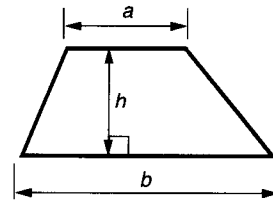
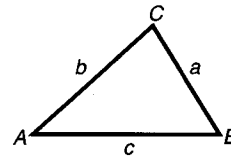**Volume of prism** = (area of cross-section) × length

In any triangle *ABC*

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
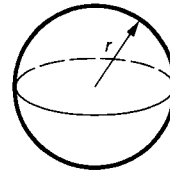
Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

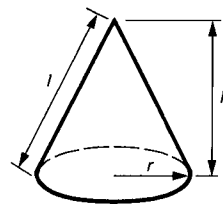**Area of triangle** $= \frac{1}{2}ab\sin C$

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

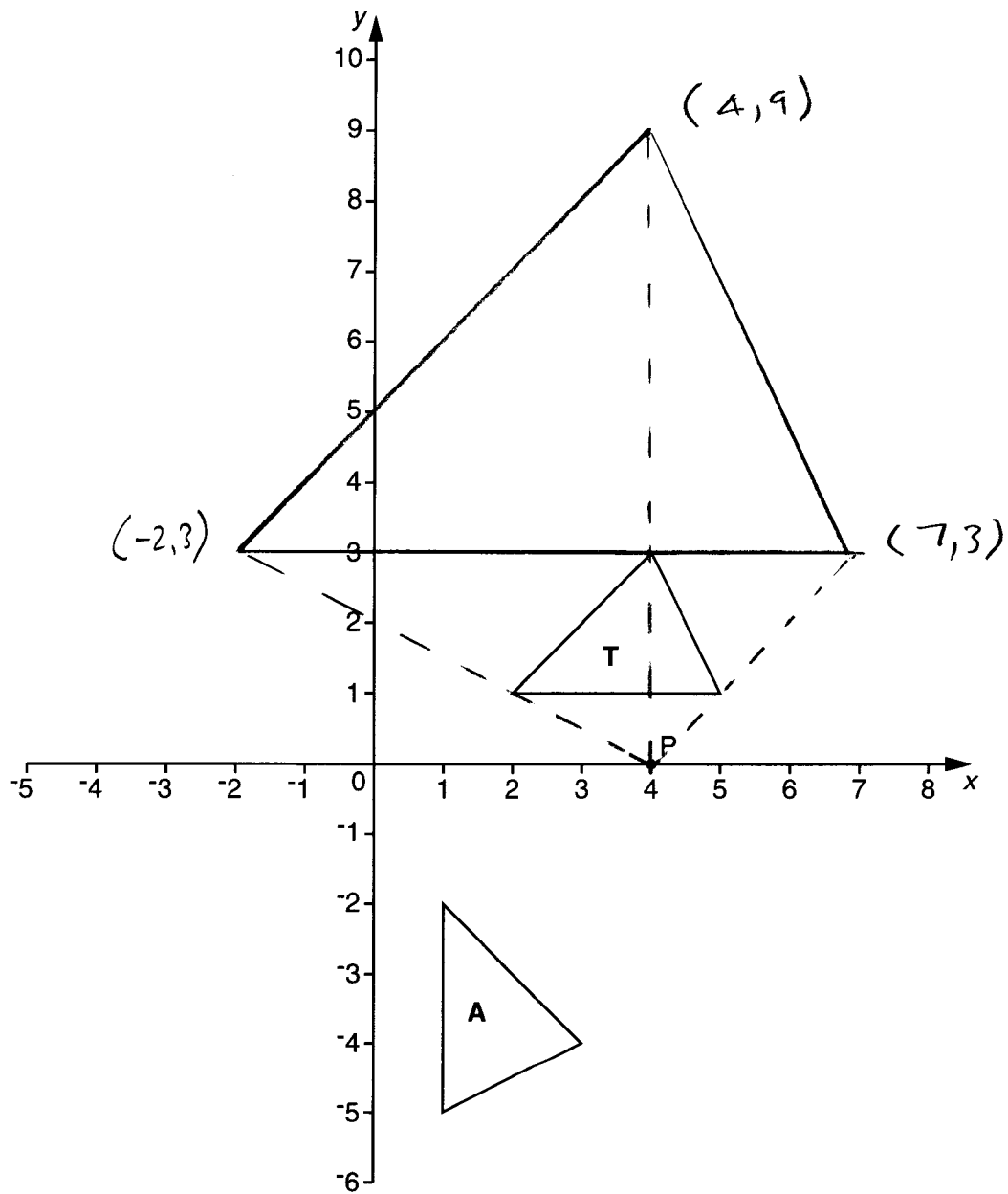**Curved surface area of cone** $= \pi r l$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

1 Here is a grid with two triangles, **T** and **A**.

$(4, 9)$

$(-2, 3)$

$(7, 3)$

T

P

A

(a) Describe fully the **single** transformation that maps triangle **T** onto triangle **A**.

Rotation by 90° clockwise about (0, 0)

_____ [3]

(b) Enlarge triangle **T** with scale factor 3 and centre P (4, 0). [2]

　　　　　　　　　　　　　　　　　　　　　　　　　　Turn over

2   This is a graph for converting Pounds (£) to Danish Kroner (DKK).
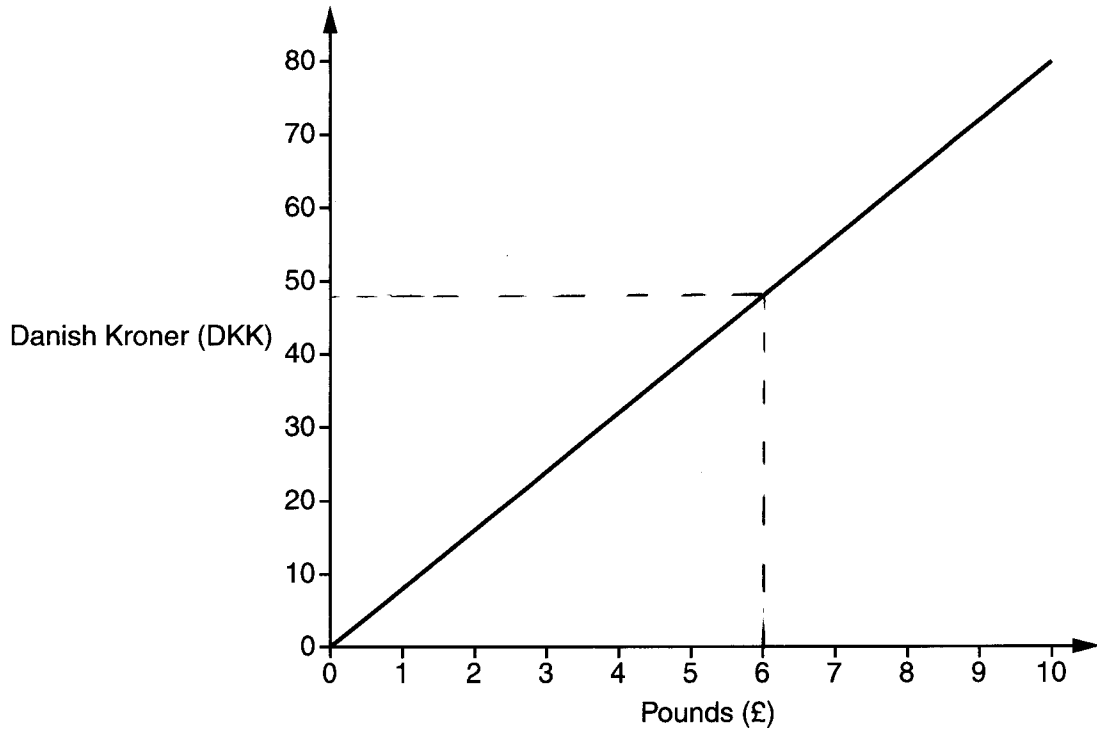
(a)   Use the graph to convert £6 to Danish Kroner (DKK).

(a) _____ 48 _____ DKK [1]

(b)   Work out the gradient of the line.

$$\frac{80 - 0}{10 - 0} = 8$$

(b) _____ 8 _____ [2]

(c)   Explain what this gradient represents.

_____ The number of DKK in £1 _____ [1]

(d)   Convert 152 DKK to Pounds.

$$\frac{152}{8} = £19$$

(d) £ _____ 19 _____ [2]

3 **(a)** Here is a list of numbers.

$$39 \quad 43 \quad 57 \quad 79 \quad 91 \quad 111$$

Write down all the numbers in this list which are prime numbers.

(a) $43, \; 79$ [1]

(b) Write 42 as a product of its prime factors.

$$\begin{array}{c|c} 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

(b) $42 = 2 \times 3 \times 7$ [2]

(c) Find the lowest common multiple of 24 and 42.

$$\begin{array}{c|c} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$24 = 2 \times 2 \times 2 \times 3$

$42 = 2 \times 3 \times 7$

$LCM = 2 \times 2 \times 2 \times 3 \times 7 = 168$

(c) 168 [2]

(d) A travel firm has to take 95 pupils on a visit.
It has taxis which take 7 passengers and minibuses which take 15 passengers.
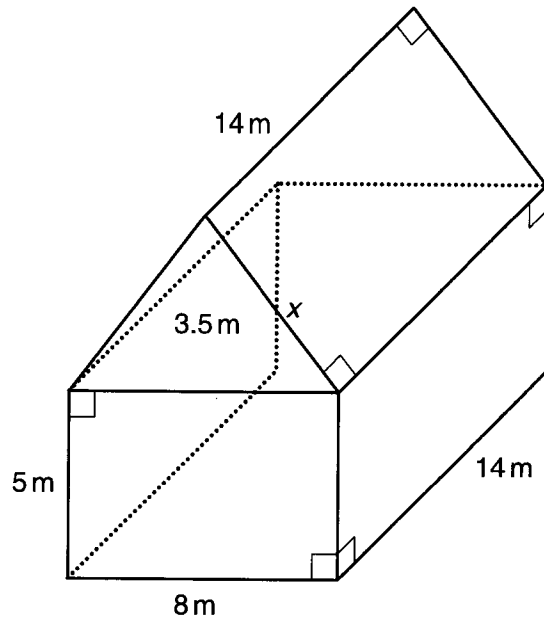They do not want to have any empty seats.

Work out how many taxis and minibuses they need to use.

$$5 \times 7 = 35$$
$$4 \times 15 = \underline{60}$$
$$95$$

(d) taxis = 5

minibuses = 4 [2]

Turn over
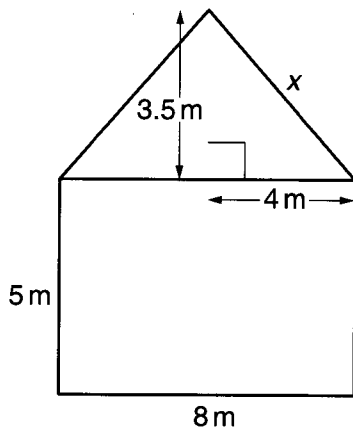
4   Here is a diagram of a barn.

(a) The front elevation of the barn is sketched below.

Calculate the length x.

Not to scale

$$x^2 = 3.5^2 + 4^2$$

$$x^2 = 12.25 + 16$$

$$x^2 = 28.25$$

$$x = \sqrt{28.25}$$

$$x = 5.315$$

(a) _____ 5.3 _____ m **[3]**

(b) (i) Draw the **plan view** of the barn on the grid below using a scale of 1 cm to 1 m.

4cm

4cm

8 cm

14cm
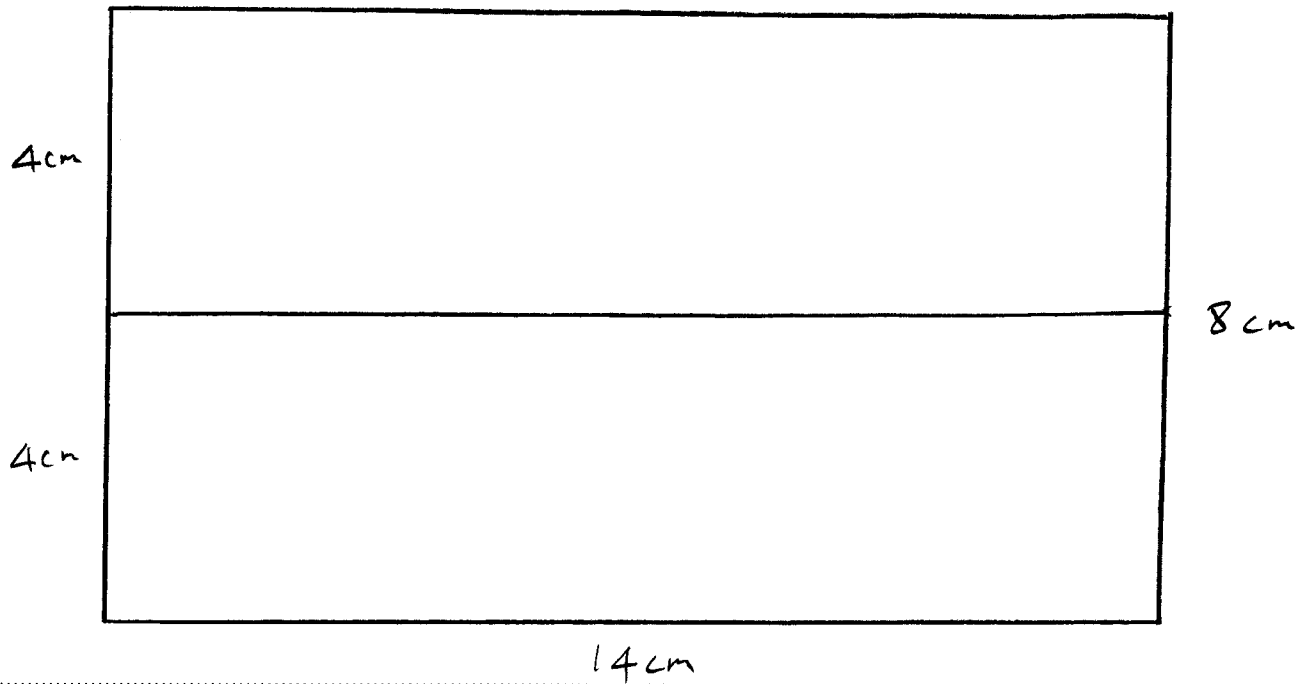
[1]

(ii) Draw the **side elevation** of the barn on the grid below using a scale of 1 cm to 1 m.

3.5cm

5cm

14 cm

[1]

　　　　　　　　　　　　　　　　　　　　　　　　　　Turn over
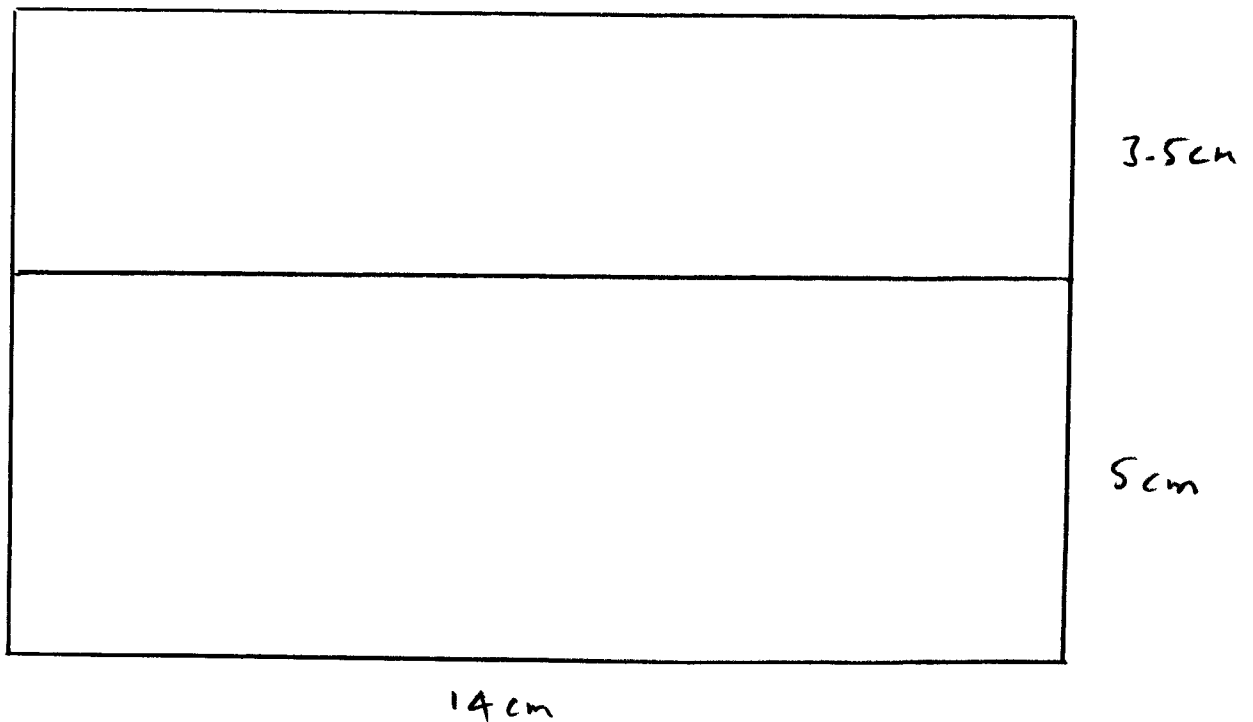
5   Here are the first four terms of a sequence.

6   12   18   24

17   23   29   35

Rule +6

Write an expression for the nth term.

$n^{th} \ term \ = \ 6n + 11$

$6n + 11$

_____ [2]

6   (a)   Multiply out the brackets and simplify.

$5(x - 3) + 2(x + 5)$

$= \ 5x - 15 + 2x + 10$

$= \ 7x - 5$

(a) _____ $7x - 5$ _____ [2]

(b)   Solve.

$12x - 11 = 4x + 9$

$12x - 4x = +9 + 11$

$8x = 20$

$x = \dfrac{20}{8}$

$x = 2\frac{4}{8}$

$x = 2\frac{1}{2}$

(b) $x =$ _____ $2\frac{1}{2}$ _____ [3]

7   Golf scores are recorded on cards.
    The table summarises the scores for one day.

Score	Frequency
60 – 66	10
67 – 73	15
74 – 80	14
81 – 87	4

Midpoint   Freq × Midpoint
63   630
70   1050
77   1078
84   336
            _____
            3094

43

(a)   Calculate an estimate of the mean score.

$$\text{Estimate of mean} = \frac{3094}{43}$$
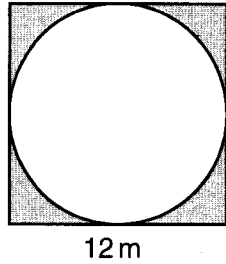
$$= 71.95$$

(or 72)
allowed

(a) _____ 71.95 _____ [4]

(b)   A card is picked at random.

Work out the probability that the score on the card is 73 or below.

(or 0.58)

$$\frac{25}{43}$$

(b) _____ $\frac{25}{43}$ _____ [2]

Turn over

8   The diagram shows a circular pond with paving stones around the edge making up a square.
    The length of each side of the square is 12 m.

Not to scale

12 m

Calculate the shaded area.

Circle has diameter 12m so radius is 6m

Area of circle $= \pi r^2 = \pi \times 6^2 = 113.1 \, m^2$

Area of square $= 12 \times 12 = 144 \, m^2$

Shaded Area $= 144 - 113.1 = 30.9 \, m^2$

_____30.9_____ m² [4]

9 **(a)** Calculate.

$$\sqrt{18.5^2 - 11.1^2}$$

$$= \sqrt{(18.5^2 - 11.1^2)}$$

$$= 14.8$$

(a) _____ 14.8 _____ [1]

(b) Here are three cards.

A	B	C
$\dfrac{1}{(2.5^2 - 1.5^2)}$	$\left(\dfrac{35}{54}\right)^2$	$\sqrt[3]{0.06}$

Work out the values written on each card.
Put the values in order, smallest first.

0·25   0.42   0·39

(b) _____ A _____ _____ C _____ _____ B _____ [2]

        smallest

Turn over

10 (a) The equation $x^3 - x^2 - 40 = 0$ has a solution between $x = 3$ and $x = 4$.

Find this value of x correct to 1 decimal place.
Show clearly your trials and the values of their outcomes.

x			
3.5	$3.5^3 - 3.5^2 - 40$	-9.375	Too small
3.8	$3.8^3 - 3.8^2 - 40$	0.432	Too big
3.7	$3.7^3 - 3.7^2 - 40$	-3.037	Too small
3.75	$3.75^3 - 3.75^2 - 40$	-1.328	Too small

So round up to 3.8

(a) $x =$ ___3.8___ [3]

(b) Solve.

$$\frac{(x-5)}{3} + \frac{(3x+4)}{2} = 15$$

$$\frac{6(x-5)}{3} + \frac{6(3x+4)}{2} = 6 \times 15$$

$$2(x-5) + 3(3x+4) = 90$$

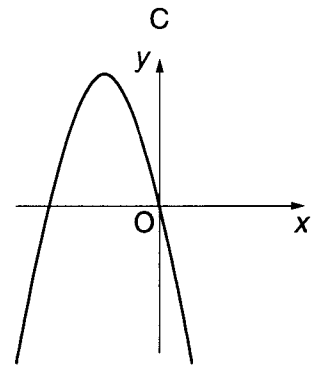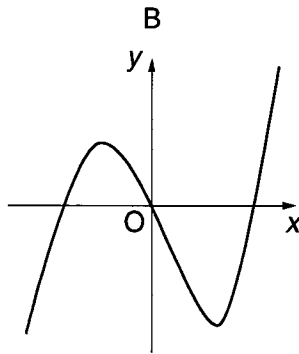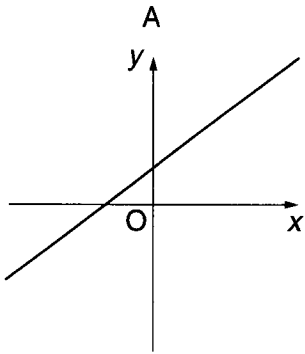$$2x - 10 + 9x + 12 = 90$$

$$11x + 2 = 90$$

$$11x = 90 - 2$$

$$11x = 88$$

$$x = \frac{88}{11}$$

$$x = 8$$

(b) $x =$ ___8___ [4]

11 Here are three sketch graphs.

A	B	C

Write the equation of each graph in the spaces on the answer line.
Choose your answers from this list.

$y = {}^-4x - 4x^2$ $y = 4x$ $y = x^3 - 4x + 4$ $y = 4x^2 - 4x$

$y = {}^-4x + 4$ $y = x^3 - 4x$ $y = 4x - 4x^2$ $y = x + 4$

Graph A is $y =$ __$x + 4$__

Graph B is $y =$ __$x^3 - 4x$__

Graph C is $y =$ __$-4x - 4x^2$__ [3]

12 (a) Write 16 000 in standard form.

(a) 1.6×10^4 _____ [1]

(b) Here are some facts about four planets.

	Mercury	Venus	Earth	Mars
Mass (kg)	3.30×10^{23}	4.87×10^{24}	5.97×10^{24}	6.42×10^{23}
Volume (m³)	6.08×10^{19}	9.28×10^{20}	1.08×10^{21}	1.63×10^{20}

(i) Complete this sentence, giving your answer correct to 3 significant figures.

$$\frac{9.28 \times 10^{20}}{6.08 \times 10^{19}} = 15.263$$

The volume of Venus is ___ 15.3 ___ times the volume of Mercury. [2]

(ii) Show that the Earth has the greatest density.
Make all your working clear. [3]

$$Density = \frac{Mass}{Volume}$$

Mercury
$$\frac{3.30 \times 10^{23}}{6.08 \times 10^{19}}$$
5428

Venus
$$\frac{4.87 \times 10^{24}}{9.28 \times 10^{20}}$$
5248

Earth
$$\frac{5.97 \times 10^{24}}{1.08 \times 10^{21}}$$
5528

Mars
$$\frac{6.42 \times 10^{23}}{1.63 \times 10^{20}}$$
3939

$\frac{kg}{m^3}$

So Earth has greatest density of $5528 \frac{kg}{m^3}$

13 Make c the subject of this formula.

$$E = mc^2$$

$$\frac{E}{m} = c^2$$

$$\sqrt{\frac{E}{m}} = c$$

$c =$ _____ $\sqrt{\dfrac{E}{m}}$ _____ **[2]**

14 y is directly proportional to x^2 and $y = 80$ when $x = 4$.

Write a formula for y in terms of x.

$$y = kx^2$$

Sub
$y = 80$
$x = 4$

$$80 = k \times 4^2$$
$$80 = 16k$$
$$\frac{80}{16} = k$$
$$5 = k$$

$$\Rightarrow \quad y = 5x^2$$

_____ $y = 5x^2$ _____ **[3]**

Turn over

15 Mr Chalmers gave a GCSE paper to all the 32 pupils in his class.
The results are summarised in this cumulative frequency graph.

Q3

Cumulative
frequency

Q1
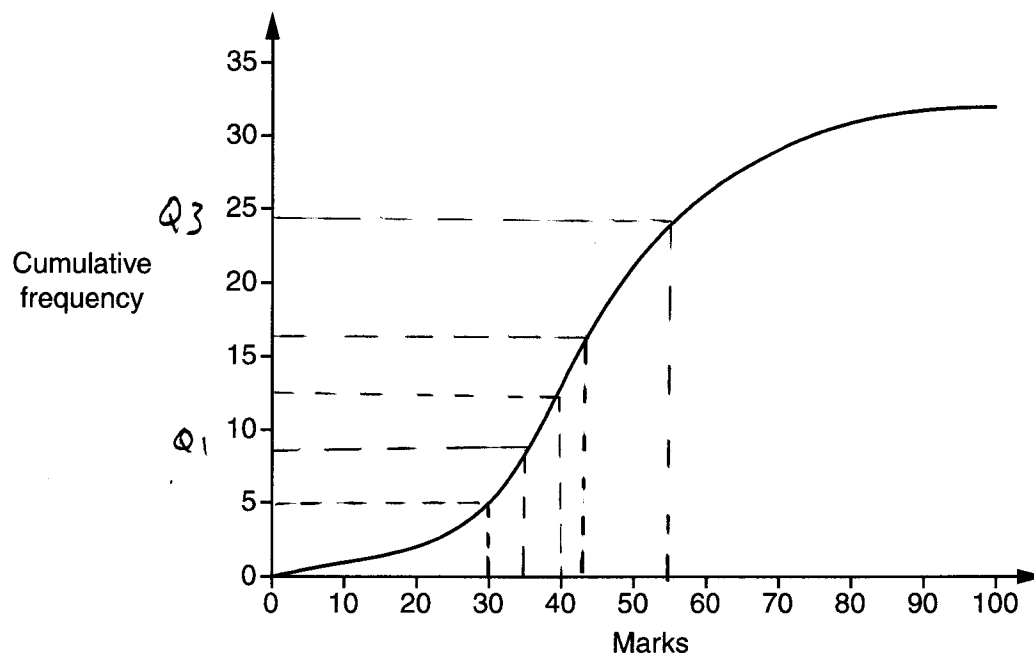
Marks

(a) Use the graph to find

   (i) the number of pupils who scored 30 marks or fewer,

(a)(i) _____ 5 _____ **[1]**

   (ii) the median,

$\left(\begin{array}{c}42-44\\allowed\end{array}\right)$

(ii) _____ 43 _____ **[1]**

   (iii) the interquartile range.

$$I\,QR = Q_3 - Q_1 = 55 - 35 = 20$$

$\left(\begin{array}{c}20-22\\allowed\end{array}\right)$

(iii) _____ 20 _____ **[2]**

(b)* The marks for each grade for the GCSE paper are given in the table below.

Mark	Grade
0 to 9	U
10 to 24	E
25 to 40	D
41 to 54	C
55 to 69	B
70 to 84	A
85 to 100	A*

The percentage of students nationally achieving a grade C, or better, for the paper was 55%. Mr Chalmers says that his pupils' results are better than this.

Is he correct?
Show your working clearly.

From graph 13 pupils scored up to 40

So 32 - 13 = 19 pupils scored more than 40 to obtain a Grade C or better

$$\frac{19}{32} \times 100 = 59.4\% \text{ of the class}$$

So yes Mr Chalmers' class results were better than the 55% achieved nationally

_____

_____

_____

_____ **[5]**

(c) Explain why this may not be a sensible comparison.

May not be a random sample of students.
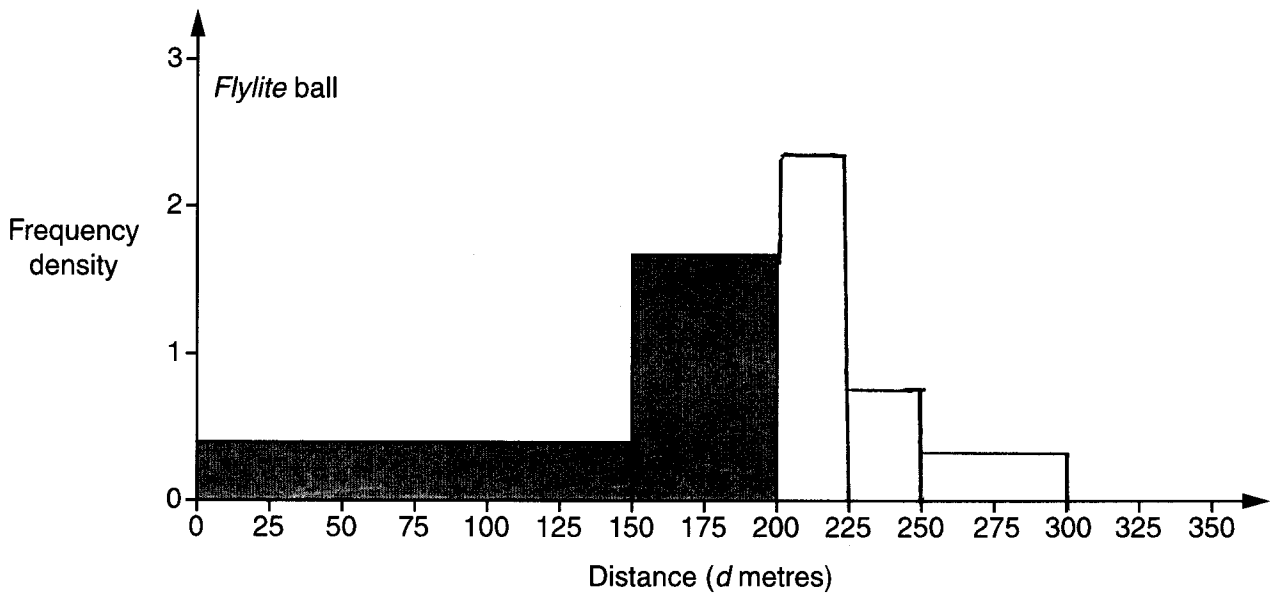It could be a top set. **[1]**

Turn over

16 A golfer records the distances he hits golf balls.

(a) The table shows the distances with *Flylite* balls.

Distance (d metres)	$0 \leqslant d < 150$	$150 \leqslant d < 200$	$200 \leqslant d < 225$	$225 \leqslant d < 250$	$250 \leqslant d < 300$
Frequency	60	84	58	20	15

Complete the histogram for this information.
The shaded values have been drawn for you.

[2]

$$Frequency \ Density = \frac{Frequency}{Interval \ Width}$$

$$\frac{58}{25} = 2.32 \qquad \frac{20}{25} = 0.8 \qquad \frac{15}{50} = 0.3$$

(b) The histogram below summarises the distances with the *Arrowe* balls.

Distance (*d* metres)
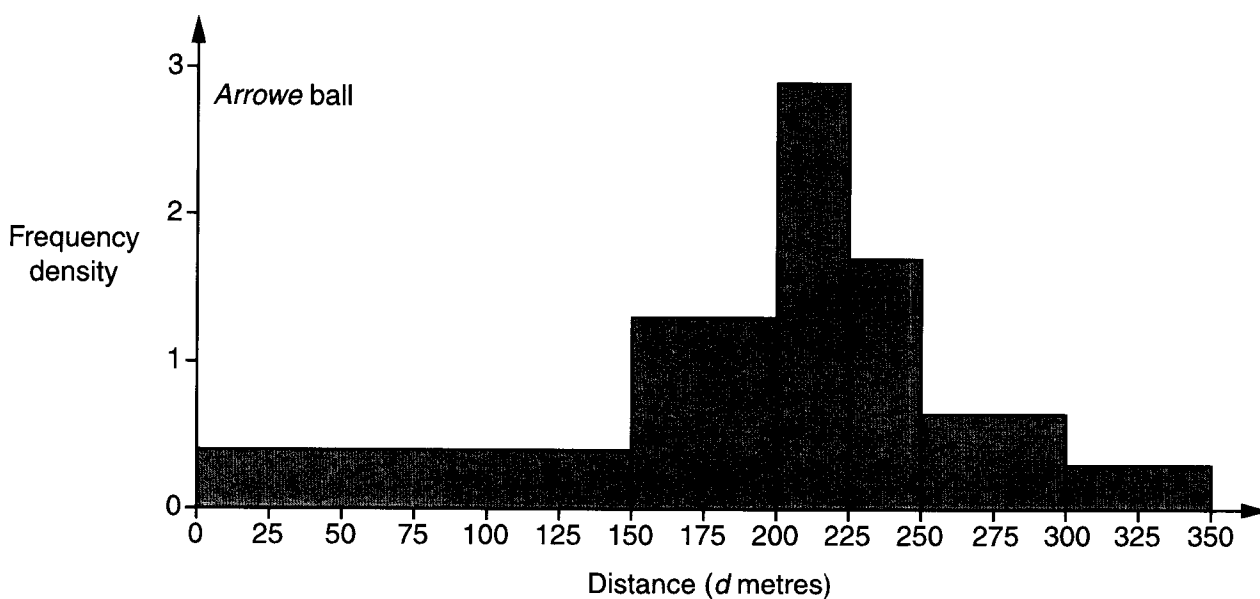
Make two different comments comparing the distances he hits these two types of ball.
Calculations are not necessary.

Comment 1

On average the Arrowe balls travel further than
the Flylite balls

Comment 2

Only the Arrowe balls are hit in excess of
300m

[2]

Turn over

17 Here are the equations of two graphs.

$$y^2 = x^2 - 2x + 10 \quad ①$$
$$y = 3x + 2 \quad ②$$

(a) Show that the point of intersection of these graphs satisfies the equation $4x^2 + 7x - 3 = 0$.

[3]

Sub for y in ①

$$(3x + 2)^2 = x^2 - 2x + 10$$
$$(3x + 2)(3x + 2) = x^2 - 2x + 10$$
$$9x^2 + 6x + 6x + 4 = x^2 - 2x + 10$$
$$9x^2 + 12x + 4 - x^2 + 2x - 10 = 0$$
$$8x^2 + 14x - 6 = 0$$
$$4x^2 + 7x - 3 = 0$$

(b) Solve the equation $4x^2 + 7x - 3 = 0$, giving your answers correct to 2 decimal places.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 4 \times (-3)}}{2 \times 4}$$

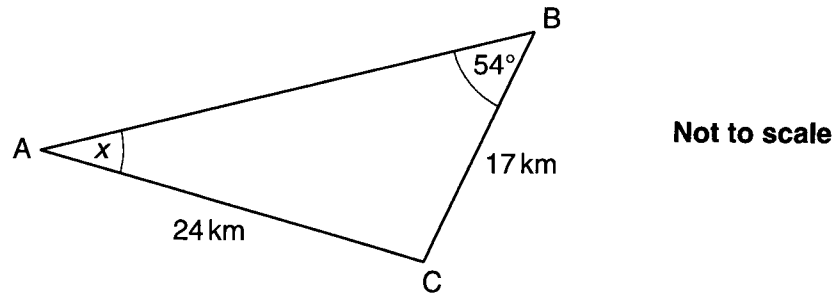$$x = \frac{-7 \pm \sqrt{49 + 48}}{8}$$

$$x = \frac{-7 \pm \sqrt{97}}{8}$$

$$x = \frac{-7 + \sqrt{97}}{8} = 0.3561$$

$$x = \frac{-7 - \sqrt{97}}{8} = -2.1061$$

(b) x = _____0.36_____ and x = _____−2.11_____ [3]

to 2 d.p. as required

18 ABC is a triangle.

Not to scale

Calculate angle *x*.

Sine rule

$$\frac{17}{\sin x} = \frac{24}{\sin 54°}$$

$$\Rightarrow \quad \frac{\sin x}{17} = \frac{\sin 54°}{24}$$

$$\Rightarrow \quad \sin x = \frac{\sin 54}{24} \times 17$$

$$\Rightarrow \quad \sin x = 0.57305$$

$$\Rightarrow \quad x = \sin^{-1}(0.57305)$$

$$\Rightarrow \quad x = 34.963°$$

$$\underline{x = 34.96°} \quad °\text{ [3]}$$

(Allowed 34.75 − 35°)

Turn over

19 A building project is expected to cost £4 500 000.
The agreed completion date is 1 January 2014.
After this date, for every month it is delayed, the cost increases by 2% of the cost for the previous month.

(a) Calculate the cost on 1 April 2014.   1 Feb, 1 Mar, 1 Apr   3 months

$$4,500,000 \times 1.02^3$$

$$= 4,775,436$$

(a) £ __4,775,436__ [1]

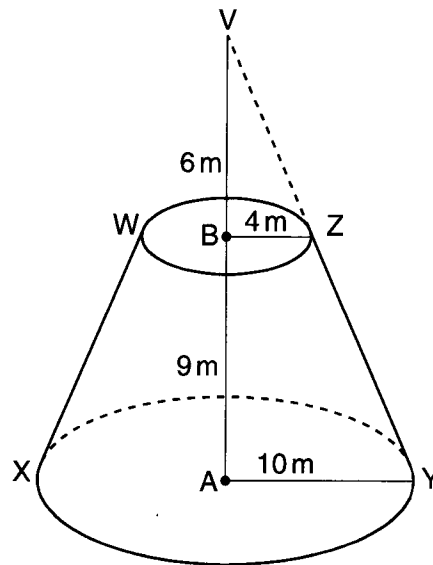**(b)** When the cost first exceeds £5 500 000, for how many months has the project been delayed?

$$4,500,000 \times 1.02^{10} = 5,485,475$$

$$4,500,000 \times 1.02^{11} = 5,595,184$$

So delayed 11 months when cost exceeds £5,500,000

(b) __11 months__ [3]

20 WXYZ is a frustum of a cone.

Volume of cone

$$= \frac{1}{3}\pi r^2 h$$

The base radius, AY, of the frustum is 10 m and the top radius, BZ, is 4 m.
VB = 6 m and BA = 9 m.

Calculate the volume of the frustum.

Volume = Volume of big cone − Volume of little cone

$$= \frac{1}{3}\pi \times 10^2 \times 15 \quad - \frac{1}{3}\pi \times 4^2 \times 6$$

$$= 1570.796 \quad - \quad 100.531$$

$$= 1470.265$$
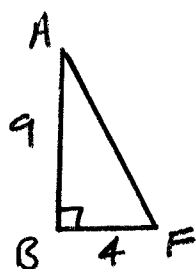
1470 _____ m³ **[4]**

TURN OVER FOR QUESTION 21

(Allowed 1469.5 − 1470.9)

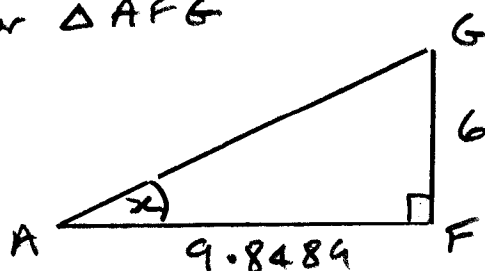**21** ABCDEFGH is a cuboid.
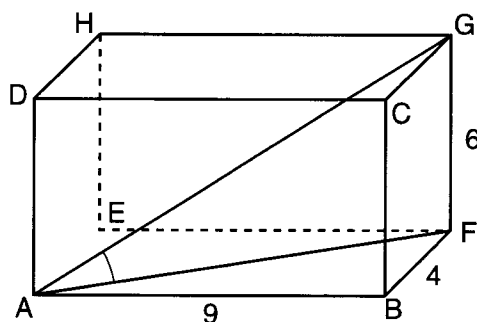
Calculate the angle GAF.

Look at base

$$AF^2 = 4^2 + 9^2$$
$$AF^2 = 16 + 81$$
$$AF^2 = 97$$
$$AF = \sqrt{97} = 9.8489$$

Now consider $\triangle AFG$

$$\tan x = \frac{6}{9.8489} \implies x = \tan^{-1}\left(\frac{6}{9.8489}\right)$$
$$\implies x = 31.3^\circ$$

$$\angle GAF = 31.3^\circ$$

_____ ° **[5]**

END OF QUESTION PAPER

$$\left(Allowed\ 31.3 - 31.5^\circ\right)$$