

Wednesday 6 November 2013 – Morning

GCSE MATHEMATICS B

J567/03 Paper 3 (Higher Tier)

Solutions

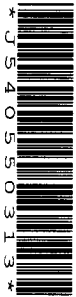
Candidates answer on the Question Paper.

OCR supplied materials:
None

Duration: 1 hour 45 minutes

Other materials required:

- Geometrical instruments
- Tracing paper (optional)



Candidate forename		Candidate surname	
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Centre number										Candidate number				
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INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Your answers should be supported with appropriate working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- Your quality of written communication is assessed in questions marked with an asterisk (*).
- The total number of marks for this paper is **100**.
- This document consists of **20** pages. Any blank pages are indicated.

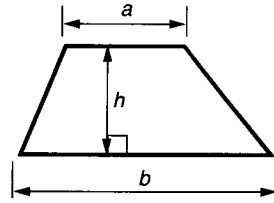
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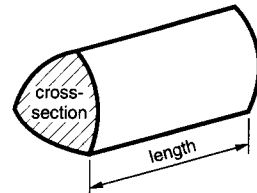
No calculator can be used for this paper

Formulae Sheet: Higher Tier

Area of trapezium = $\frac{1}{2}(a + b)h$



Volume of prism = (area of cross-section) \times length

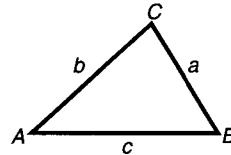


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

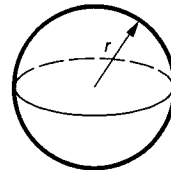
Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$



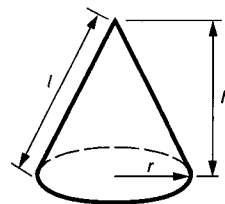
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PLEASE DO NOT WRITE ON THIS PAGE

Answer **all** the questions.

1 (a) Written as the product of its prime factors, $108 = 2^2 \times 3^3$.

(i) Write 96 as the product of its prime factors.

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \overline{)3} \\ \hline 1 \end{array}$$

$$96 = 2^5 \times 3$$

(a)(i) $2^5 \times 3$ [2]

(ii) Find the highest common factor of 96 and 108.

$$\begin{aligned} 108 &= 2^2 \times 3^3 = \textcircled{2} \times \textcircled{2} \times 3 \times 3 \times \textcircled{3} \\ 96 &= 2^5 \times 3 = \textcircled{2} \times \textcircled{2} \times 2 \times 2 \times 2 \times \textcircled{3} \\ \text{HCF} &= 2 \times 2 \times 3 = 12 \end{aligned}$$

(ii) 12 [2]

(b) Work out.

$$1\frac{3}{4} + 3\frac{5}{12}$$

Give your answer as a mixed number in its simplest form.

$$= 4 \frac{9+5}{12}$$

$$= 4 \frac{14}{12}$$

$$= 5 \frac{2}{12}$$

$$= 5 \frac{1}{6}$$

(b) $5 \frac{1}{6}$ [3]

- 2 (a) Complete the following sentences.

The negative square root of 49 is -7.

5 is the cube root of 125.

[2]

- (b) Work out the value of $2b^2 - b$ when $b = -3$.

$$\begin{aligned} & 2(-3)^2 - (-3) \\ &= 2 \times 9 + 3 \\ &= 18 + 3 \\ &= 21 \end{aligned}$$

(b) 21 [2]

- 3 (a) Felix wants to work out how much it costs him to use his tumble dryer. The tumble dryer uses 1.9 units of electricity to dry one load of washing. Felix dries four loads of washing each week. He pays 12.8p for every unit of electricity he uses.

Work out the weekly cost, in pence, of using the tumble dryer.

$$\begin{array}{r} 1.9 \\ \underline{4} \times \\ 7.6 \end{array}$$

$$12.8 \times 7.6$$

$$\begin{array}{r} 12.8 \\ \underline{7.6} \times \\ 768 \\ 8960 \\ \hline 97.28 \end{array}$$

OR

	100	20	8	
70	7000	1400	560	7000
6	600	120	48	1400
				560
				600
				120
				48 +
				<u>97.28</u>

(a) 97.28 p [4]

or 97p to nearest penny

- (b) Felix reads the electricity meter in his house each week.
Here is his record of the number of units he has used each week for 16 weeks.

74 73 86 78 70 74 73 60
64 67 49 77 53 68 57 54

- (i) Complete the stem and leaf diagram to represent this data.

4 | 9
5 | 3 7 4
6 | 0 4 7 8
7 | 4 3 8 0 4 3 7
8 | 6

4	9
5	3 4 7
6	0 4 7 8
7	0 3 3 4 4 7 8
8	6

Key: 7|4 represents 74

[3]

- (ii) Felix reads in a newspaper article that the average household uses 330 units of electricity each month.

How does the electricity he uses compare with this figure?
Support your answer using Felix's data.

16 items median is item $\frac{16+1}{2}$ 8.5th
8th is 68
9th is 70
median = 69

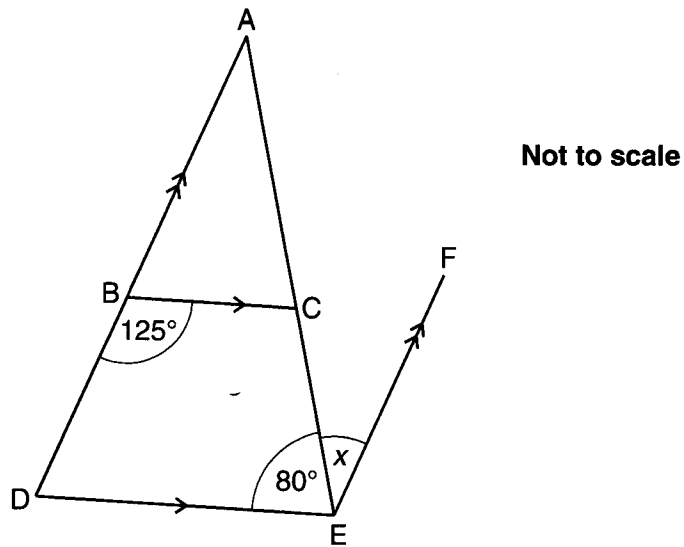
Taking a month to be 4 weeks

$$69 \times 4 = 276 \text{ units}$$

Felix uses substantially less than the average
of 330 units per month

[4]

- 4* In the diagram ADE is a triangle.
BC is parallel to DE and DBA is parallel to EF.



Work out angle x .
Give a reason for each step of your working.

[5]

$$\angle BDE = 180 - 125 = 55^\circ$$

(Allied angles add up to 180°)

$$\angle DEF = 180 - 55 = 125^\circ$$

(Allied angles add up to 180°)

$$x = 125 - 80 = 45^\circ$$

(Together the x and the 80° make $\angle DEF$)

- 5 (a) Wanda asked a random sample of 120 students from a college what they were planning to do when they left college.
The table shows the results of Wanda's survey.

University	Apprenticeship	Job
74	16	30

- (i) One of these students is selected at random.

What is the probability that the student is planning to go to university?

(a)(i) $\frac{74}{120}$ [1]

- (ii) There are 2400 students at the college.

Estimate the number of these students who plan to get a job.

$$2400 \times \frac{30}{120}$$

$$= 2400 \times \frac{1}{4}$$

$$= 600$$

(ii) 600 [2]

- (b) Wanda plans to interview some of the students in more detail.
Of the 2400 students, 1500 are male and 900 are female.
She decides to interview a representative stratified sample of 80 students.

How many male students should she interview?

$$\text{Sample fraction} = \frac{80}{2400} = \frac{8}{240} = \frac{1}{30}$$

$$\text{Male sample size} = 1500 \times \frac{1}{30} = \frac{1500}{30} = \frac{150}{3} = 50$$

(b) 50 [2]

- 6 (a) Solve this inequality.

$$3x - 4 \leq 8$$

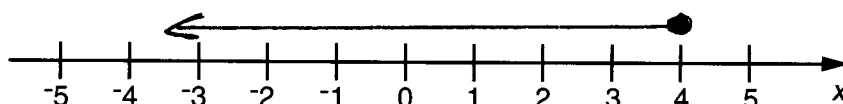
$$3x \leq 8 + 4$$

$$3x \leq 12$$

$$x \leq \frac{12}{3} \quad x \leq 4$$

(a) $x \leq 4$ [2]

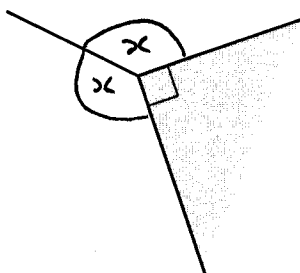
- (b) Represent your solution on the number line.



[1]

- 7 A floor is tiled using a pattern of two different shaped tiles. One of the shapes is a square and the other is a regular polygon.

At each vertex in the pattern, two of the polygon tiles and one square tile meet.



What shape is the regular polygon?
Show your reasoning clearly.

Find interior angle x

$$2x + 90 = 360$$

$$2x = 360 - 90$$

$$2x = 270$$

$$x = \frac{270}{2}$$

$$x = 135^\circ$$

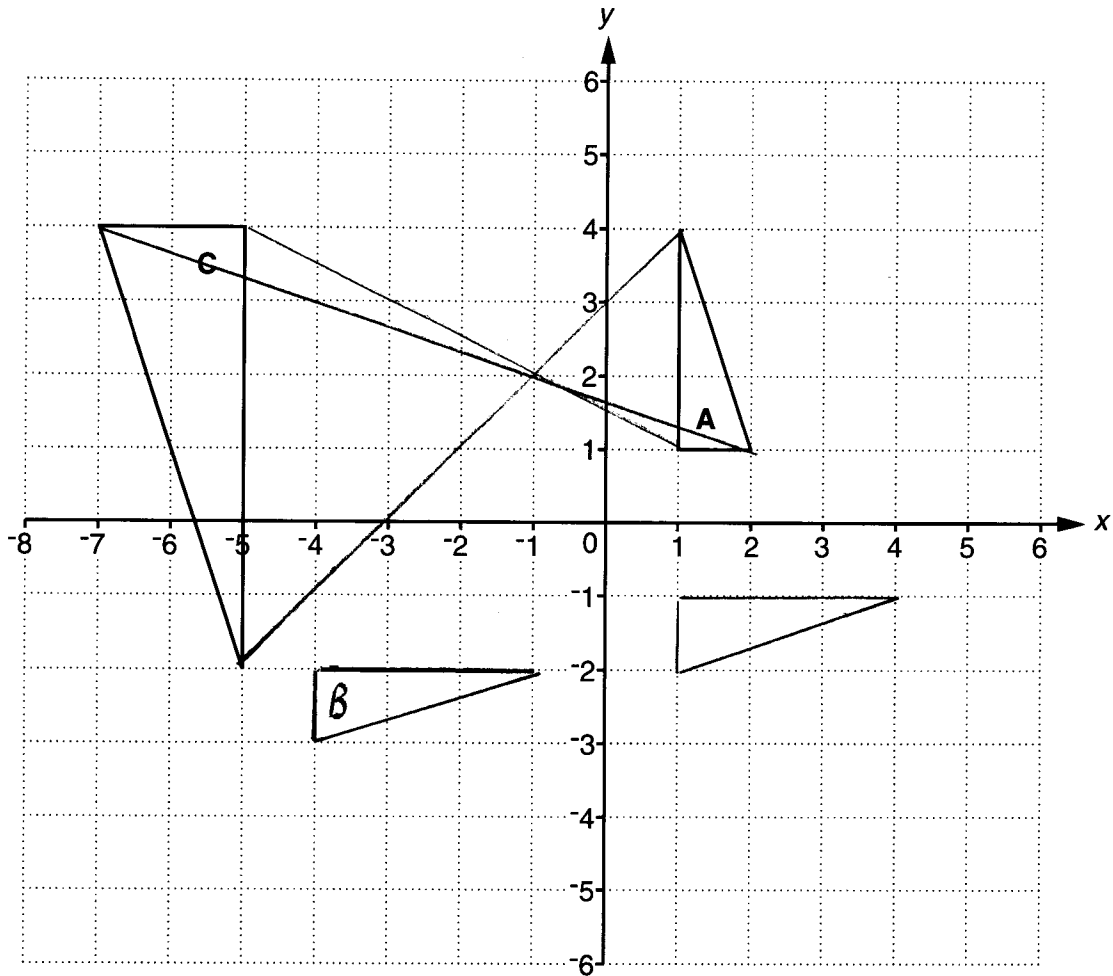
(Angles at a point
add up to 360°)

$$\text{Exterior Angle} = 180 - x = 180 - 135 = 45^\circ$$

$$\frac{360}{45} = 8 \text{ so polygon is an octagon (exterior angle} = \frac{360}{\text{number of sides}})$$

[4]

8 Triangles **A** and **C** are drawn on the grid below.



(a) Triangle **B** is the image of triangle **A** after:

- a rotation of 90° clockwise about the origin, followed by
- a translation of $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$.

Draw and label triangle **B** on the diagram.

[4]

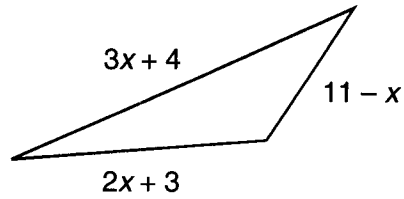
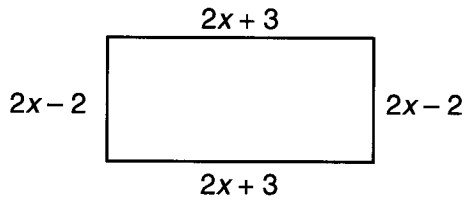
(b) Triangle **C** is an enlargement of triangle **A**.

Describe the enlargement fully.

Enlargement scale factor -2 about centre of
enlargement $(-1, 2)$

[2]

- 9 The perimeter of this rectangle is equal to the perimeter of this triangle.



Not to scale

Find the length of the shortest side of the triangle.

$$2x+3 + 2x+3 + 2x-2 + 2x-2 = 3x+4 + 2x+3 + 11-x$$

$$8x + 2 = 4x + 18$$

$$8x - 4x = 18 - 2$$

$$4x = 16$$

$$x = \frac{16}{4}$$

$$x = 4$$

$$\text{Sides of } \Delta \quad 3(4)+4 = 16$$

$$2(4)+3 = 11$$

$$11-4 = 7$$

Shortest side = 7 [5]

- 10 A group of 60 students entered a general knowledge competition. This table shows the distribution of their scores.

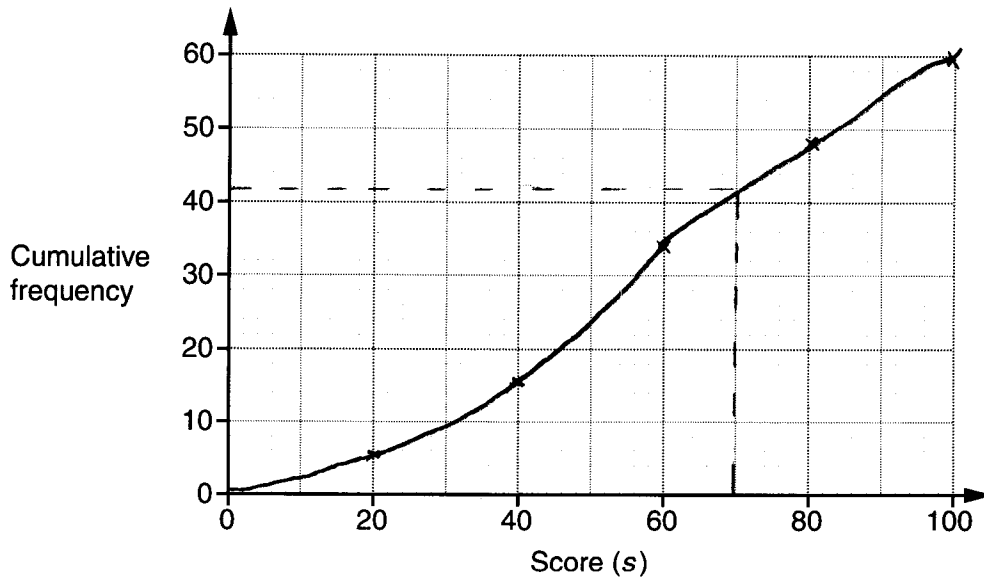
Score (s)	$0 < s \leq 20$	$20 < s \leq 40$	$40 < s \leq 60$	$60 < s \leq 80$	$80 < s \leq 100$
Frequency	5	10	19	14	12

- (a) Complete the cumulative frequency table for this distribution.

Score (s)	$s \leq 20$	$s \leq 40$	$s \leq 60$	$s \leq 80$	$s \leq 100$
Cumulative frequency	5	15	34	48	60

[1]

- (b) Draw a cumulative frequency graph to represent the distribution.



[2]

- (c) Students scoring **more than** 75 won a prize. Use the graph to estimate the number of students who won a prize.

42 less than 75 so $60 - 42 = 18$ more than 75

(c) 18 [2]

11 The table below shows the number of barrels of oil produced each day in some countries.

Country	Oil produced (barrels per day)
Russia	1.01×10^7
Saudi Arabia	9.76×10^6
USA	9.06×10^6
China	3.99×10^6

- (a) How many barrels per day were produced in the USA?
Give your answer as an ordinary number.

(a) 9,060,000 [1]

- (b) How many more barrels per day were produced in Russia than in Saudi Arabia?

$$\begin{array}{r} 9101 \\ 10,100,000 \\ - 9,760,000 \\ \hline 340,000 \end{array}$$

(b) 340,000 [2]

- (c) The production of one barrel per day is equivalent to approximately 50 tonnes of oil per year.

Estimate the number of tonnes of oil produced in China each year.
Give your answer in standard form.

$$\begin{aligned} & 3.99 \times 10^6 \times 50 \\ &= 3.99 \times 10^6 \times \frac{100}{2} \\ &= \frac{3.99 \times 10^8}{2} \\ &= 1.995 \times 10^8 \end{aligned}$$

(c) 1.995×10^8 tonnes [2]

- 12 A garden pond has a surface area of 3m^2 .
The pond is a prism of constant depth 60cm .

$$3 \times 0.6 = 1.8$$

- (a) (i) Calculate the volume, in m^3 , of the pond.

(a)(i) 1.8 m^3 [2]

- (ii) How many litres of water does the pond contain?

$$1 \text{ litre} = 1000 \text{ cm}^3$$

$$1.8 \text{ m}^3 = 1.8 \times 100 \times 100 \times 100 \text{ cm}^3 = 1,800,000$$

$$\frac{1,800,000}{1000} = 1800$$

(ii) 1800 litres [1]

- (b) A scale model of the pond is made.
The depth of the pond in the model is 6cm .

$$\begin{aligned} \text{Length} & 6:60 \\ & = 1:10 \end{aligned}$$

$$\begin{aligned} \text{Area} & = 1:10^2 \\ & = 1:100 \end{aligned}$$

- (i) Find the surface area of the model pond.

$$\text{Area} = \frac{3\text{m}^2}{100} = 0.03\text{m}^2$$

(b)(i) 0.03 m^2 [3]

- (ii) How many litres of water does the model pond contain?

$$0.03 \times 0.06 = 0.0018 \text{ m}^3$$

$$= 0.0018 \times 100 \times 100 \times 100 \text{ cm}^3$$

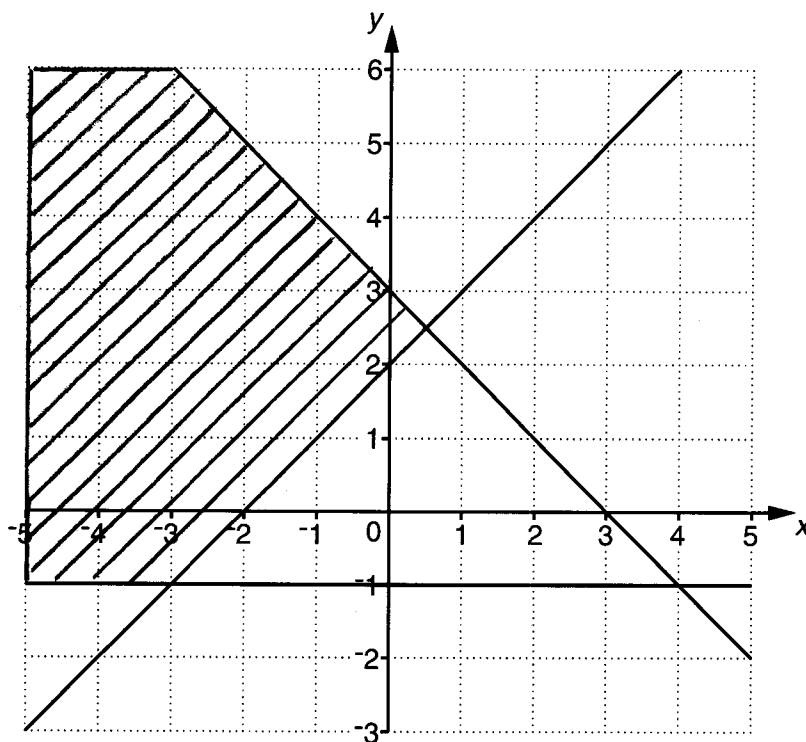
$$= 1800 \text{ cm}^3$$

(ii) 1.8 litres [1]

$$1 \text{ litre} = 1000 \text{ cm}^3$$

$$\text{so } \frac{1800}{1000} = 1.8 \text{ litres}$$

- 13 The graphs of $y = -1$, $y = x + 2$ and $x + y = 3$ are drawn on the grid below.



Show clearly the region **R** which satisfies all of these inequalities.

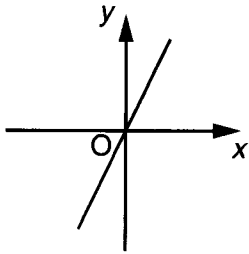
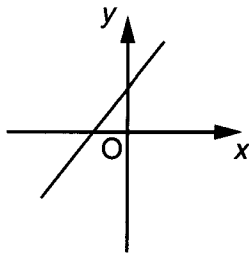
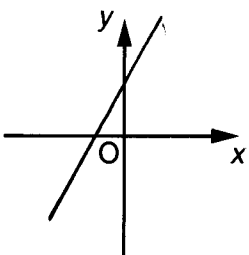
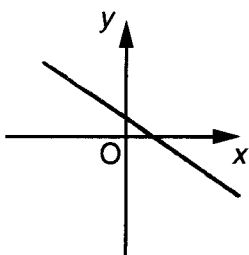
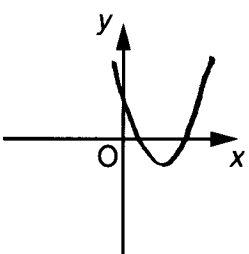
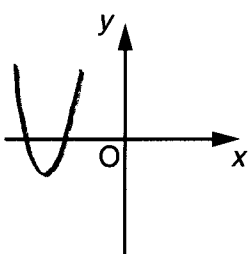
$$y \geq -1 \quad x + y \leq 3 \quad y \geq x + 2$$

[3]

- 14 Each statement below is sometimes true and sometimes false.

For each statement sketch a graph to show an example where it is true and an example where it is false.

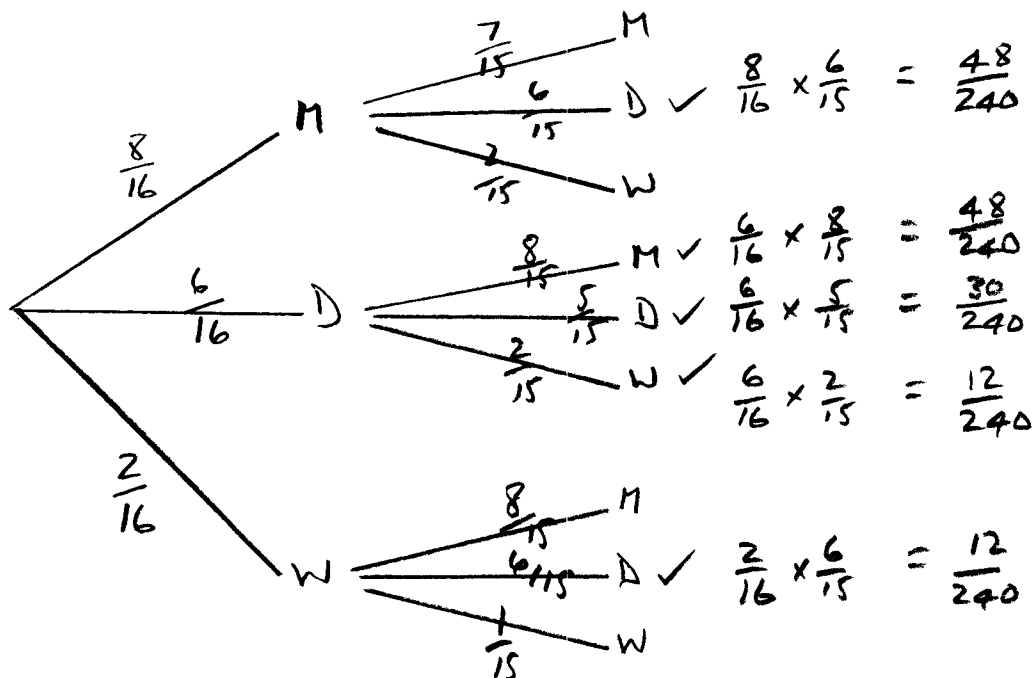
The first one has been done for you.

Statement	True	False
A straight line graph goes through the origin.		
The gradient of a straight line graph is positive.		
A quadratic equation $ax^2 + bx + c = 0$ has two positive solutions.		

[3]

- 15 A box contains 8 milk chocolates, 6 dark chocolates and 2 white chocolates. Varun takes a chocolate from the box at random and eats it. He then takes another chocolate at random and eats it.

Find the probability that Varun eats at least one dark chocolate.



$$\text{Prob (At least one dark)} = \frac{48 + 48 + 30 + 12 + 12}{240} = \frac{150}{240}$$

$$\frac{150}{240}$$

[4]

$$\text{or } \frac{15}{24}$$

$$\text{or } \frac{5}{8}$$

16 (a) Solve.

$$\frac{3x-1}{5} = x-2$$

$$3x-1 = 5(x-2)$$

$$3x-1 = 5x-10$$

$$3x-5x = -10+1$$

$$-2x = -11$$

$$x = \frac{-11}{-2} \quad x = +5\frac{1}{2}$$

(a) $x =$ 5 $\frac{1}{2}$ [3]

(b) Simplify.

$$\left(\frac{a^5}{a^9}\right)^{-2} = \left(\frac{a^9}{a^5}\right)^2 = \frac{a^{18}}{a^{10}} = a^8$$

(b) a⁸ [2]

(c) Express as a single fraction in its simplest form.

$$\frac{4}{x-2} - \frac{5}{x+1} = \frac{4(x+1) - 5(x-2)}{(x-2)(x+1)}$$

$$= \frac{4x+4-5x+10}{(x-2)(x+1)}$$

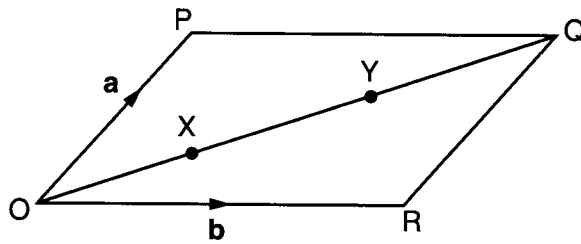
$$= \frac{14-x}{(x-2)(x+1)}$$

(c) $\frac{14-x}{(x-2)(x+1)}$ [3]

17 OPQR is a parallelogram.

$$\vec{OP} = \mathbf{a} \text{ and } \vec{OR} = \mathbf{b}.$$

X and Y are the points on OQ such that $OX = XY = YQ$.



Not to scale

Find as simply as possible in terms of \mathbf{a} and \mathbf{b} ,

(a) \vec{OQ} ,

$$(a) \quad \underline{\mathbf{a}} + \underline{\mathbf{b}} \quad [1]$$

(b) \vec{PR} ,

$$(b) \quad \underline{-\mathbf{a}} + \underline{\mathbf{b}} \quad [1]$$

(c) \vec{XY} ,

$$(c) \quad \underline{\frac{1}{3}(\mathbf{a} + \mathbf{b})} \quad [1]$$

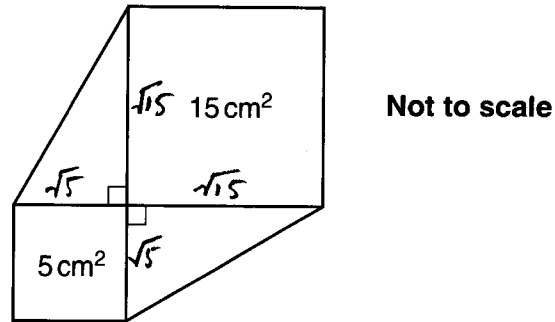
(d) \vec{XR} .

$$\begin{aligned} &= \underline{\vec{OX}} + \underline{\vec{OR}} \\ &= \underline{\frac{1}{3}(\mathbf{a} + \mathbf{b})} + \underline{\mathbf{b}} \\ &= \underline{\frac{1}{3}\mathbf{a}} + \underline{\frac{1}{3}\mathbf{b}} + \underline{\mathbf{b}} \end{aligned}$$

$$(d) \quad \underline{\hspace{10em}} \quad [2]$$

$$= \underline{-\frac{1}{3}\mathbf{a}} + \underline{\frac{2}{3}\mathbf{b}}$$

- 18 The diagram shows a shape made from two squares and two right-angled triangles. The area of the smaller square is 5 cm^2 and the area of the larger square is 15 cm^2 .



Work out the area of the complete shape.

Give your answer in the form $a + b\sqrt{c}$, where c is as small as possible.

$$\begin{aligned} \text{Each } \Delta \text{ has area } \frac{1}{2} \text{ base} \times \text{height} &= \frac{1}{2} \times \sqrt{5} \times \sqrt{15} \\ &= \frac{1}{2} \times \sqrt{75} \end{aligned}$$

$$\begin{aligned} \text{Total area} &= 15 + 5 + \frac{1}{2}\sqrt{75} + \frac{1}{2}\sqrt{75} \\ &= 20 + \sqrt{75} \\ &= 20 + \sqrt{25 \times 3} \\ &= 20 + 5\sqrt{3} \text{ cm}^2 \end{aligned}$$

$$\underline{20 + 5\sqrt{3}} \text{ cm}^2 \text{ [4]}$$

TURN OVER FOR QUESTION 19

19 Solve algebraically these simultaneous equations.

$$y = 4x^2 - 9x - 1 \quad (1)$$

$$y = 5 - 4x \quad (2)$$

Sub for y in (1)

$$5 - 4x = 4x^2 - 9x - 1$$

$$0 = 4x^2 - 9x - 1 + 4x - 5$$

$$0 = 4x^2 - 5x - 6$$

$$0 = 4x^2 + 3x - 8x - 6$$

$$0 = x(4x + 3) - 2(4x + 3)$$

$$0 = (x - 2)(4x + 3)$$

Either $x - 2 = 0$

$$x = 2$$

When $x = 2$

$$y = 5 - 4(2)$$

$$y = 5 - 8$$

$$y = -3$$

$$(2, -3)$$

or $4x + 3 = 0$

$$4x = -3$$

$$x = -\frac{3}{4}$$

when $x = -\frac{3}{4}$

$$y = 5 - 4\left(-\frac{3}{4}\right)$$

$$y = 5 + 3$$

$$y = 8$$

$$\left(-\frac{3}{4}, 8\right)$$

$$x = \underline{\quad 2 \quad} \quad y = \underline{\quad -3 \quad}$$

$$x = \underline{\quad -\frac{3}{4} \quad} \quad y = \underline{\quad 8 \quad} \quad [6]$$

$$4x - 6$$

$$= -24$$

factors $+1 \quad -24$

$$-1 \quad +24$$

$$+2 \quad -12$$

$$-2 \quad +12$$

$$+3 \quad -8 \quad \checkmark$$

$$-3 \quad +8$$

$$+4 \quad -6$$

$$-4 \quad +6$$

END OF QUESTION PAPER

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