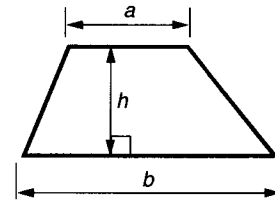


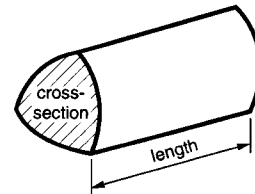


## Formulae Sheet: Higher Tier

**Area of trapezium** =  $\frac{1}{2}(a + b)h$



**Volume of prism** = (area of cross-section)  $\times$  length

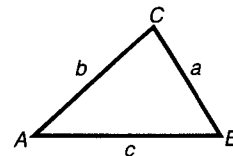


**In any triangle ABC**

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

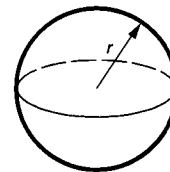
**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2}ab \sin C$



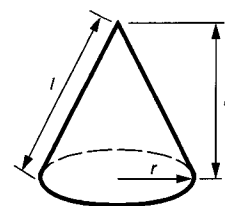
**Volume of sphere** =  $\frac{4}{3}\pi r^3$

**Surface area of sphere** =  $4\pi r^2$



**Volume of cone** =  $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



**The Quadratic Equation**

The solutions of  $ax^2 + bx + c = 0$ ,  
where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**PLEASE DO NOT WRITE ON THIS PAGE**

Answer all the questions.

- 1 (a) Calculate.

$$\left(\frac{6.3^2 - 3.7}{5.8}\right) = 6.20517$$

Write your answer correct to 2 decimal places.

(a) 6.21 [2]

- (b) Calculate.

$$\sqrt{4.5 \times 6.7 + 1.8 \times 2.4} = \sqrt{34.47}$$

Write your answer correct to 2 significant figures.

$$= 5.8711$$

(b) 5.9 [2]

- 2 Samuel has six types of coin in a bag.  
The table shows the probability of each type of coin being picked.

Coin	1p	2p	5p	10p	20p	50p
Probability	0.07	0.23	0.18	0.28	0.19	x

- (a) Work out x.
- $$0.07 + 0.23 + 0.18 + 0.28 + 0.19 = 0.95$$
- $$1 - 0.95 = 0.05$$

(a) 0.05 [2]

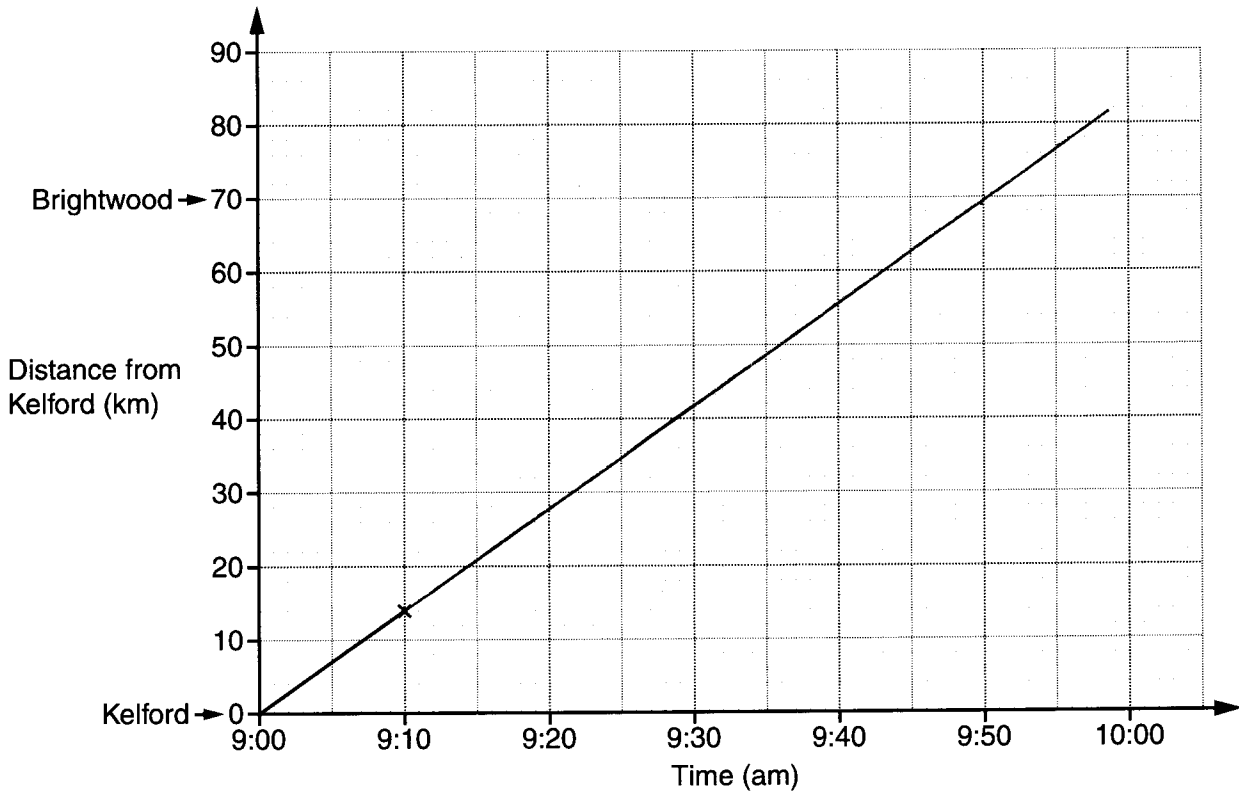
- (b) Samuel picks one coin out of the bag at random.

Work out the probability that he picks a coin worth 5p or less.

$$0.07 + 0.23 + 0.18 = 0.48$$

(b) 0.48 [2]

- 3 A train travels from Kelford to Brightwood.  
The graph shows the first ten minutes of the train's journey.



The two stations are 70 kilometres apart.  
The train is due to arrive at Brightwood at 10:00 am.

Will it arrive on time if it continues to travel at the same speed?  
Show clearly how you decide.

Yes - same speed continues same slope  
so arrives about 9:50

---



---

[3]

- 4 (a) Here are the first four terms of a sequence.

5	10	15	20
7	12	17	22

Write an expression for the  $n$ th term of this sequence.

(a)            $5n + 2$            [2]

- (b) The  $n$ th term of another sequence is given by the expression  $100 - 8n$ .

Write down the first three terms of this sequence.

$$100 - 8 \times 1 = 92$$

$$100 - 8 \times 2 = 84$$

$$100 - 8 \times 3 = 76$$

(b)           92          ,          84          ,          76           [2]

- 5 Solve.

$$6(2x - 3) = 24$$

$$12x - 18 = 24$$

$$12x = 24 + 18$$

$$12x = 42$$

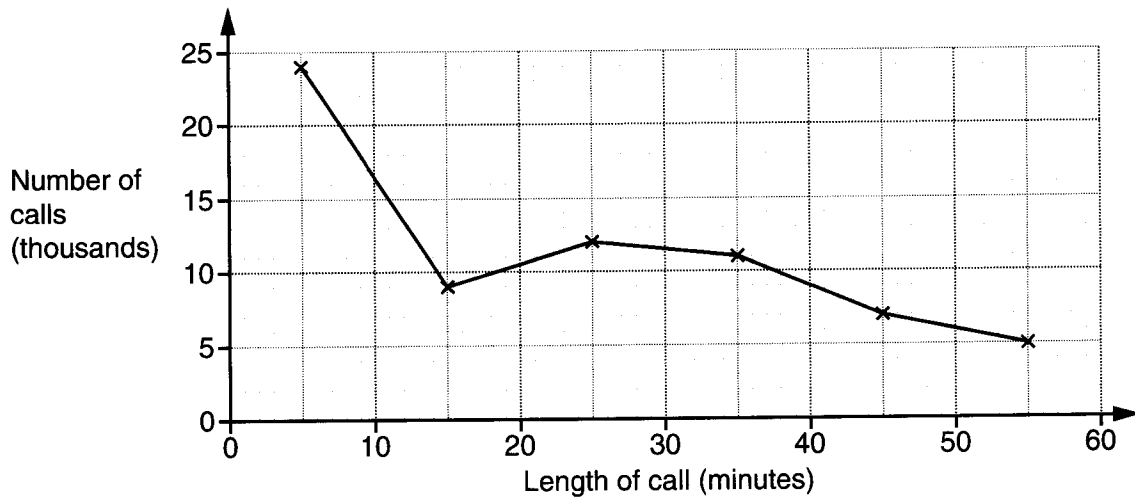
$$x = \frac{42}{12}$$

$$x = 3\frac{6}{12}$$

$$x = 3\frac{1}{2}$$

$x =$             $3\frac{1}{2}$            [3]

- 6 (a) The Bilberry Telephone Company records the lengths of telephone calls in one day. The information is summarised in the frequency polygon below.



- (i) Estimate how many calls lasted less than 20 minutes.

$$24000 + 9000 = 33000$$

(a)(i) 33 thousand [2]

- (ii) Write down the modal class.

(ii) 0 - 10 minutes [1]

- (b) The lengths of Desmond's telephone calls, in minutes, are summarised in the table below.

Length of call ( $t$ minutes)	Number of calls	Mid point	Freq $\times$ midpoint
$0 < t \leq 10$	0	5	0
$10 < t \leq 20$	3	15	45
$20 < t \leq 30$	3	25	75
$30 < t \leq 40$	6	35	210
$40 < t \leq 50$	8	45	360
$50 < t \leq 60$	5	55	275
<b>TOTALS</b>	<b>25</b>		<b>965</b>

Calculate an estimate of the mean length of Desmond's calls.

$$\text{est mean} = \frac{965}{25} = 38.6$$

(b) 38.6 minutes [4]

- (c) The table below summarises the lengths, in minutes, of Harriet's calls in November and December.

	Mean	Range
November	34.2	67.4
December	39.7	43.8

- (i) In which month were Harriet's calls longer on average?  
Explain how you decide.

December because mean is 5.5 mins greater

[1]

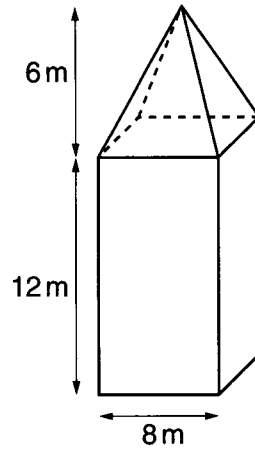
- (ii) In which month were the lengths of Harriet's calls more spread out?  
Explain how you decide.

November because range is bigger by 23.6 mins

[1]

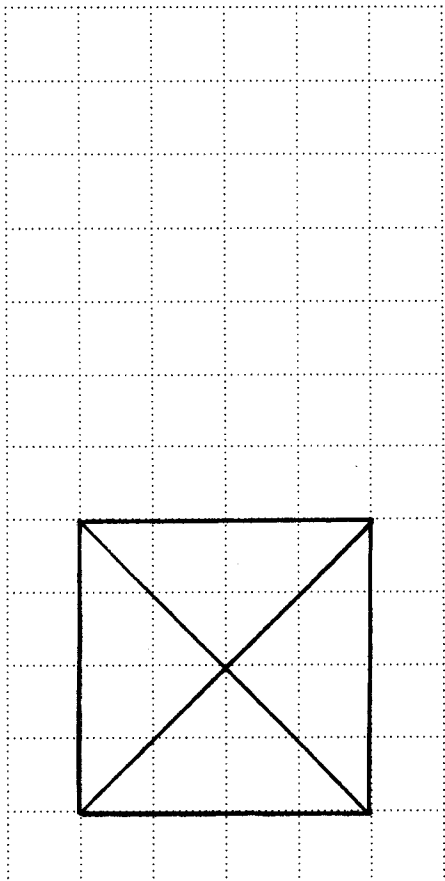
8

- 7 A tower is in the shape of a cuboid with a pyramid on top.  
The base of the tower is a square of side 8m and it has a **total** height of 18m.

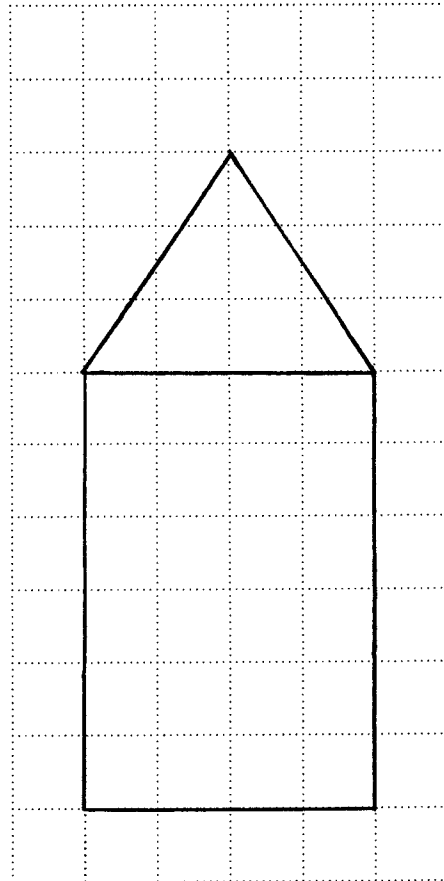


On the grids below draw accurately the plan and the front elevation of the tower.  
Use a scale of 1 cm to 2m.

Plan



Front Elevation



[4]



- 8 (a) Riverside Tennis Club has 24 members.  
They have four types of membership.

SM Senior Male  
SF Senior Female

JM Junior Male  
JF Junior Female

The membership information is recorded below.

SM	JM	SM	JM	SF	JM	SM	JF	SM 8
JM	SF	JF	SM	SM	JF	SF	SM	JM 7
SF	SM	JM	JM	JF	SM	JM	SF	SF 5
								JF 4

- (i) On the grid below, design and draw a two-way table to show this information.

		GENDER		
		MALE	FEMALE	Totals
AGE	SENIOR	8	5	13
	JUNIOR	7	4	11
Totals		15	9	24

[3]

- (ii) One member is selected at random.

Write down the probability that the member is a Junior.

(a)(ii)  $\frac{11}{24}$  [1]

- (b) In 2011, Greenmeadows Tennis Club had 25 members and in 2012 it had 31 members.

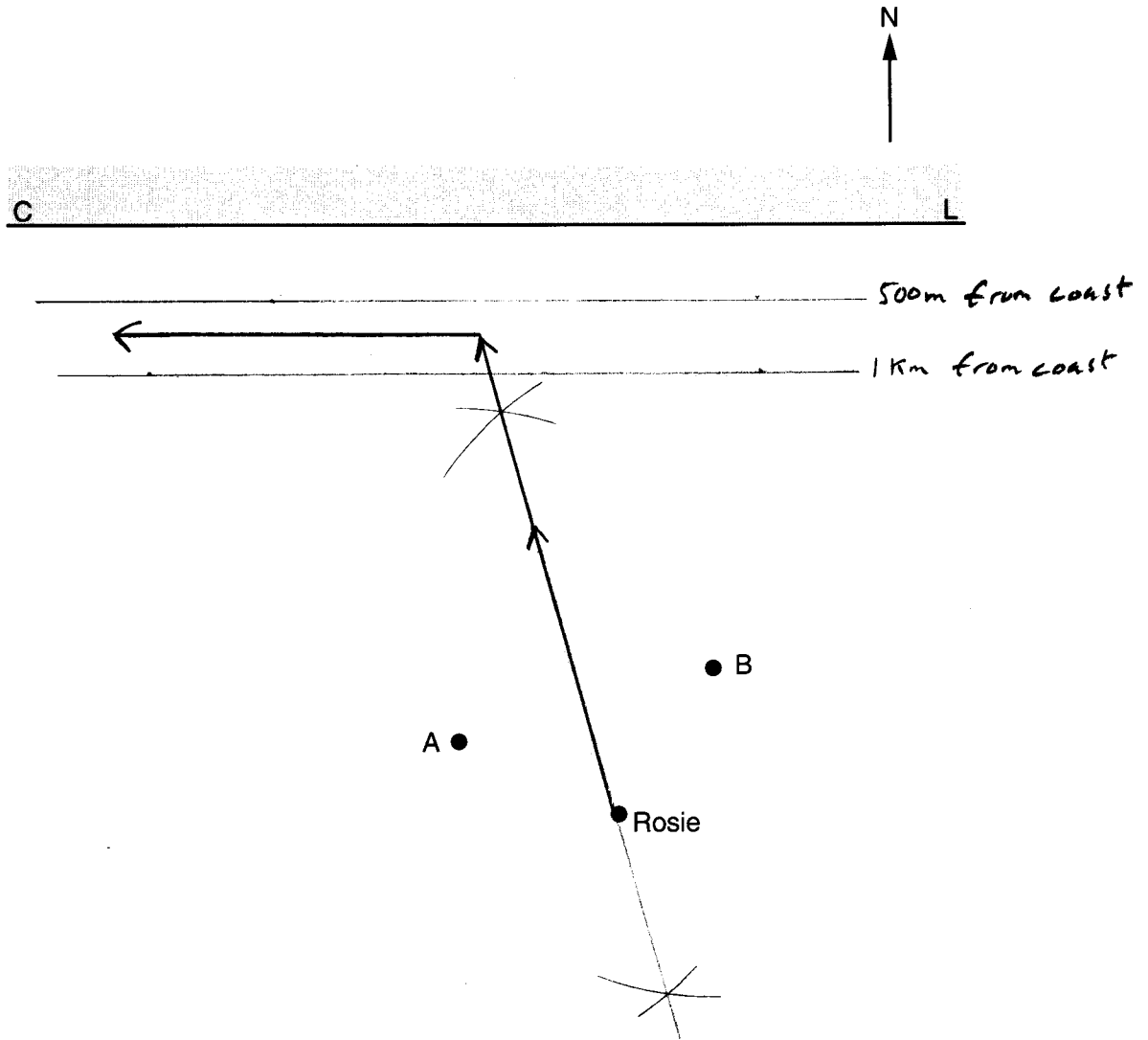
Calculate the percentage increase in the number of members.

Increase  $31 - 25 = 6$

% increase  $\frac{6}{25} \times 100 = 24\%$

(b)  $24\%$  [3]

- 9 The diagram shows a coastline, CL.  
A and B are two rocks in the sea.



Scale: 1 cm represents 500 m

Rosie is sailing her boat.  
She sails on a course towards the coast so that she is an equal distance from the rocks, A and B.

When she is less than 1 km from the coast she turns and sails due West.  
She now sails so that she is between 500 m and 1 km from the coast.

Construct a route that Rosie could take.  
You must leave in all your construction lines.

[4]

- 10 Gwen is taking her class of 28 pupils to a pantomime.  
The total cost of the trip is £575.

Use estimation to find an approximate cost of this trip for each pupil.  
Show your working clearly.

$$\underline{600} \div \underline{30} = \underline{\pounds 20} \quad [2]$$

- 11 Here are six equations of straight lines, each labelled with a letter.

<b>A</b>
$y = 4x - 7$

<b>B</b>
$y = 3x + 14$

<b>C</b>
$y = 2x + 5$

<b>D</b>
$y = -3x + 1$

<b>E</b>
$y = 14x - 7$

<b>F</b>
$y = 4x + 3$

Choose the correct letters to make each statement true.

Line   E   is the steepest line.

Lines   A   and   F   are parallel.

Lines   A   and   E   meet on the y-axis.

[3]

12 In Westercote, house prices rose by 6% from 2010 to 2011.

(a) On 1 January 2010 a house was priced at £180 000.

Calculate its price on 1 January 2011.

$$£180,000 \times 1.06 = £190,800$$

(a) £ 190,800 [3]

(b) On 1 January 2011 another house was priced at £371 000.

Calculate its price on 1 January 2010.

$$£371,000 \div 1.06 = £350,000$$

(b) £ 350,000 [3]

13 (a) Multiply out and simplify.

$$(x+7)(x-3)$$

$$x^2 + 7x - 3x - 21$$

$$x^2 + 4x - 21$$

(a)            $x^2 + 4x - 21$            [2]

(b) Factorise fully.

$$6xy - 12x^2$$

$$6x(y - 2x)$$

(b)            $6x(y - 2x)$            [2]

(c) Rearrange this formula to make  $x$  the subject.

$$A = x^2 - 4y$$

$$A + 4y = x^2$$

$$\sqrt{A + 4y} = x$$

(c)            $x = \sqrt{A + 4y}$            [2]

(d)  $y$  is inversely proportional to  $x$  and  $y = 30$  when  $x = 4$ .

Write an equation linking  $x$  and  $y$ .

$$y = \frac{k}{x}$$

$$\begin{cases} y = 30 \\ x = 4 \end{cases}$$

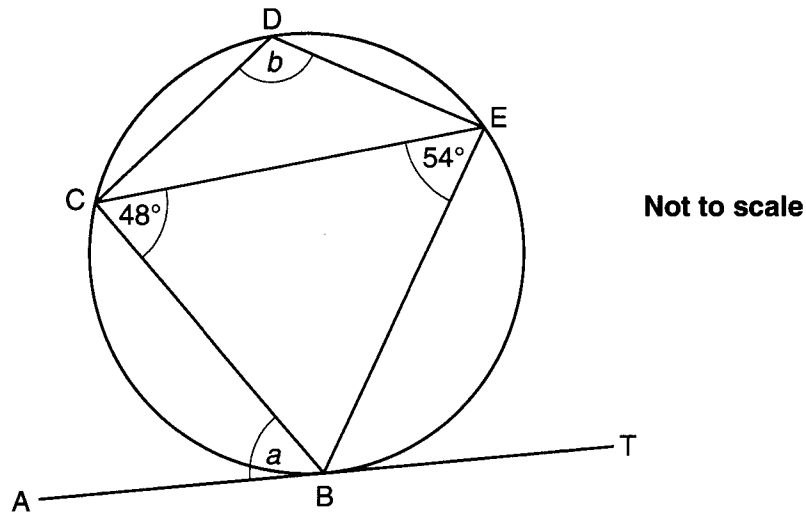
$$30 = \frac{k}{4}$$

$$30 \times 4 = k$$

$$120 = k$$

(d)            $y = \frac{120}{x}$            [3]

- 14 In the diagram B, C, D and E are points on the circumference of a circle.  
AT is the tangent to the circle at B.  
Angle BCE =  $48^\circ$  and angle BEC =  $54^\circ$ .



- (a) Find angle  $a$ .  
Give a reason for your answer.

(a) Angle  $a = \underline{54}^\circ$

Alternate segment theorem

[2]

- (b) Calculate angle  $b$ .  
Give a reason for each step of your working.

$$\begin{aligned}\angle CBE &= 180 - (48 + 54) \\ &= 180 - 102 \\ &= 78^\circ\end{aligned}$$

(angles of  $\Delta$  add to  $180^\circ$ )

$$\angle CDE = b = 180 - 78 = 102^\circ$$

(opposite angles of cyclic quad add up to  $180^\circ$ )

(b) Angle  $b = \underline{102}^\circ$  [3]

- 15 A town has a population of 120 000, correct to the nearest ten thousand, and an area of 54 km<sup>2</sup>, correct to the nearest whole number.

(a) Write down the upper bound of the population.

(a) 125,000 [1]

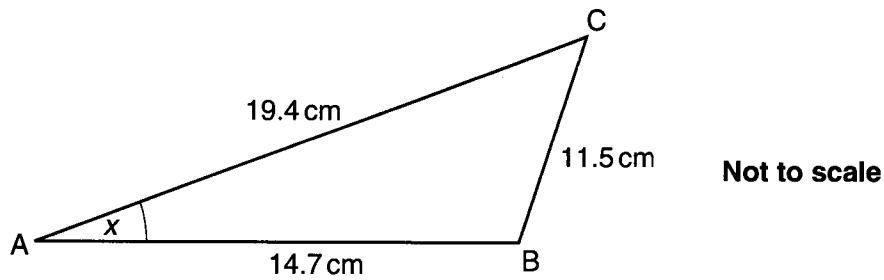
(b) Calculate the upper bound of the population density.

$$\text{Population Density} = \frac{\text{Population}}{\text{Area}}$$

$$\begin{aligned} \text{Upper bound for pop density} &= \frac{\text{Max Pop}}{\text{Min Area}} \\ &= \frac{125,000}{53.5} \end{aligned}$$

(b) 2336.45 people/km<sup>2</sup> [3]

- 16 (a) The diagram shows a triangle ABC.  
 AB = 14.7 cm, BC = 11.5 cm and AC = 19.4 cm.



- (i) Show that triangle ABC is **not** a right-angled triangle.

[3]

If right-angled then by Pythagoras

$$11.5^2 + 14.7^2 = 19.4^2$$

$$348.34 = 376.36 \quad \times \text{ Not true}$$

so triangle is not right-angled

- (ii) Calculate angle x.

Cosine rule  $a^2 = b^2 + c^2 - 2bc \cos A$

$$2bc \cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(19.4^2 + 14.7^2 - 11.5^2)}{(2 \times 19.4 \times 14.7)}$$

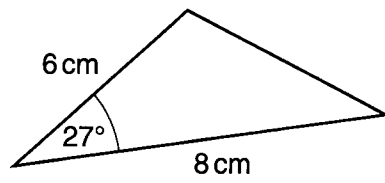
$$\cos A = 0.806859$$

$$A = \cos^{-1}(0.806859)$$

(a)(ii) 36.2 ° [3]



(b) Calculate the area of this triangle.



Not to scale

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 6 \times 8 \times \sin 27^\circ \\ &= 10.89577 \end{aligned}$$

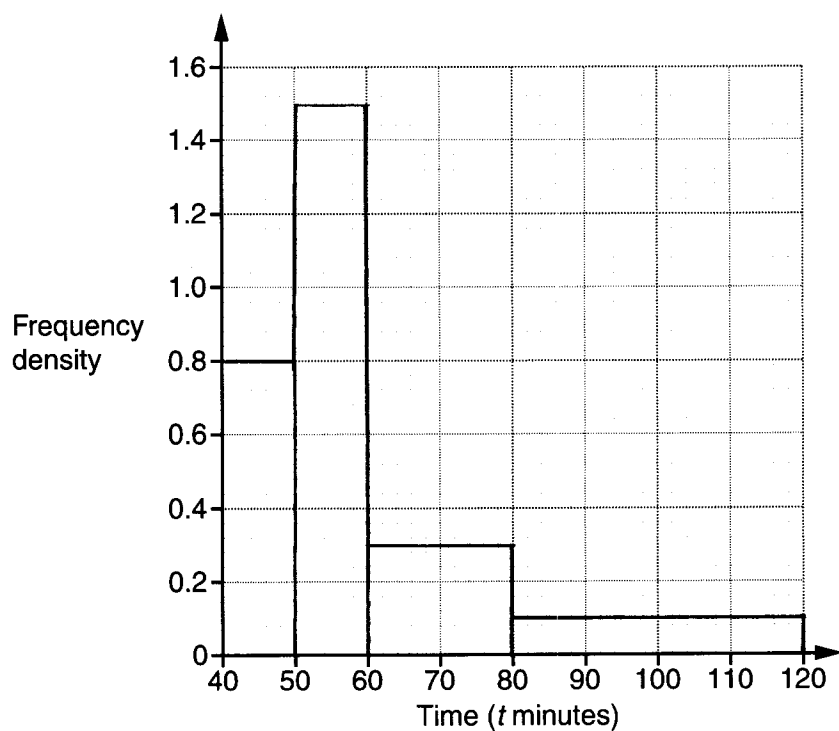
(b) 10.9 cm<sup>2</sup> [2]

- 17 A teacher records the times taken for pupils to complete a cross-country course. The results are summarised in the table below.

Time ( $t$ minutes)	Number of pupils
$40 < t \leq 50$	8
$50 < t \leq 60$	15
$60 < t \leq 80$	6
$80 < t \leq 120$	4

Freq Density  
 $8 \div 10 = 0.8$   
 $15 \div 10 = 1.5$   
 $6 \div 20 = 0.3$   
 $4 \div 40 = 0.1$

Draw a histogram on the grid below to show this data.



[3]

18 (a) Solve algebraically.

$$\begin{array}{rcl}
 5x - 2y = 22 & \textcircled{1} \\
 2x + 3y = 5 & \textcircled{2} \\
 \\ 
 \textcircled{1} \times 3 & 15x - 6y = 66 & \textcircled{3} \\
 \textcircled{2} \times 2 & 4x + 6y = 10 & \textcircled{4} \\
 \\ 
 \textcircled{3} + \textcircled{4} & 19x = 76 \\
 & x = \frac{76}{19} \\
 & x = 4
 \end{array}$$

Sub for  $x$  in  $\textcircled{2}$ 

$$2(4) + 3y = 5$$

$$8 + 3y = 5$$

$$3y = 5 - 8$$

$$3y = -3$$

$$y = \frac{-3}{3} = -1$$

$$\begin{array}{l}
 \text{(a) } x = \underline{\quad 4 \quad} \\
 y = \underline{\quad -1 \quad} \quad [4]
 \end{array}$$

(b) (i) Write  $x^2 - 6x + 4$  in the form  $(x + a)^2 + b$ .

$$= (x - 3)^2 + 4 - 9$$

$$= (x - 3)^2 - 5$$

$$\text{(b)(i) } \underline{(x - 3)^2 - 5} \quad [3]$$

(ii) Using your answer to (b)(i), or otherwise, solve  $x^2 - 6x + 4 = 0$ . Write your answers correct to 1 decimal place.

$$(x - 3)^2 - 5 = 0$$

$$(x - 3)^2 = 5$$

$$x - 3 = \pm\sqrt{5}$$

$$x = \pm\sqrt{5} + 3$$

$$x = \sqrt{5} + 3 = 5.2$$

$$\text{or } x = -\sqrt{5} + 3 = 0.8$$

$$\text{(ii) } x = \underline{5.2} \quad \text{or } x = \underline{0.8} \quad [2]$$

- 19 On Finch Island there are bullfinches and chaffinches.  
In the spring of 2013:

- the population of bullfinches was 6700 and was **decreasing** by 3% each year
- the population of chaffinches was 4800 and was **increasing** by 4% each year.

In the spring of which year will the population of chaffinches first be greater than that of the bullfinches?

Show your working clearly.

Try	Bullfinches	Chaffinches
10 yrs	$6700 \times 0.97^{10} = 4941$	$4800 \times 1.04^{10} = 7105$
7 yrs	$6700 \times 0.97^7 = 5413$	$4800 \times 1.04^7 = 6316$
6 yrs	$6700 \times 0.97^6 = 5581$	$4800 \times 1.04^6 = 6074$
5 yrs	$6700 \times 0.97^5 = 5754$	$4800 \times 1.04^5 = 5840$
4 yrs	$6700 \times 0.97^4 = 5931$	$4800 \times 1.04^4 = 5615$

Chaffinches exceed Bullfinches after 5 yrs

$$2013 + 5 = 2018$$

2018

[4]

- 20\* Assume that the Earth is a sphere with radius 6371 km.  
The land area on the surface of the Earth is 148940000 km<sup>2</sup>.

Use this information to show that the ratio of land area to water area is approximately 3 : 7. [5]

$$\begin{aligned} \text{Surface Area} &= 4\pi r^2 = 4 \times \pi \times 6371^2 \\ &= 510,064,471.9 \text{ km}^2 \end{aligned}$$

Proportion that land is of total surface

$$\begin{aligned} &= \frac{148,940,000}{510,064,471.9} \times 100 \% \\ &= 29.2 \% \end{aligned}$$

If land : water were 3 : 7

land would be  $\frac{3}{10} = 30\%$  of total surface area

$$29.2 \% \approx 30\%$$

so land to water is approximately 3 : 7

END OF QUESTION PAPER